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| PART I |

**Chapter 1**

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| Numbers |

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| 1.1 Learning objective: |

To learn about the basics of Divisibility Test & its applications, LCM & HCF, VBODMAS concepts.

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| 1.2 Introduction: |

A Number is an abstract entity that represents a count or measurement. Numbers is a very wide topic asked in placements. We will try to concentrate on few important things about Numbers.

In Hindu-Arabic Number System 0, 1, 2, 3, 4, 5, 6, 7, 8 & 9 are called as digits.

Face value: Face value is the actual value of the digit.

Example: In the number 7635, the "7" has a face value of 7; the face value of 3 is 3 and so on….

Place value: The value of where the digit is in the number, such as units, tens, hundreds, etc.

Example: In 352, the place value of the 5

is "tens" Place value of 2 \* 1 = 2;

Place value of 5 \* 10 = 50;

Place value of 3 \* 100 = 300.

**Types of Numbers**

All numbers fall into a category called Complex Numbers. Complex Numbers might be a Real Number.

A Real Number can either be a Rational number or an irrational number.

**Natural number (N):** Counting numbers are called natural numbers.

Example: 1, 2, 3,…etc. are all natural

numbers.

**Whole numbers (W):** All counting numbers together with zero from the set of whole numbers.

Example: 0, 1, 2, 3, 4,... are whole number.

**Integers (Z):**  All counting numbers, 0 and -ve of counting numbers are called integers.   
Example: -∞... -2, -1, 0, 1, 2, 3 …∞

**Rational Numbers (Q):** A Rational Number is a real number that can be written as a simple fraction.  
Example:

**Irrational Numbers:**  An Irrational Number is a real number that cannot be written as a simple fraction.   
Example: ,

**Even numbers:**  A number divisible by 2 is called an even number.   
Example: 0, 2, 4, 6 …

**Odd numbers:**A number not divisible by 2 is called an odd number.

Example: 1, 3, 5, 7 …

**Composite Numbers:** Numbers greater than 1 which are not prime are called composite numbers.  
Example: 4, 6, 8, 9, 10 …

**Prime Numbers:**A number greater than 1 having exactly two factors, namely 1 and itself is called a prime number.

Note that the number 1 is a special case which is considered to be neither composite nor prime.

**Numeric Operations:**

1. A + 0 = A ,
2. A - 0= A ,
3. A x 0 = 0 ,
4. The Value Does not exist for
5. A x 1= 1 ,
6. A + (-A) = 0 ,
7. A x (1/A) = 1
8. A + B = B + A
9. A x B = B x A
10. A ( B + C) = AB + AC

**Divisibility Test**

A number is exactly divisible by a certain divisor if on dividing the number by that divisor, the remainder is zero. On dividing a number by the divisor, we can tell whether the number is exactly divisible and if not, we can find out the remainder. In this chapter, we will learn the short-cut methods of finding whether a number is divisible by the given divisor, if not what will be the remainder on dividing the number.

**A number is divisible by:**

**2:** If unit digit of the number is either 0 or a multiple of 2 i.e. unit's digit is 0 or an even number.

Example: 18, 110 are divisible by 2. But 23, 147 are not divisible by 2.

**3:** If sum of the digits of the number is exactly divisible by 3.

Note: While totaling the digits, any digit or pairs of digits totaling to 9 may be ignored and the final sum is reduced to one digit.

Example: 132 is divisible by 3 as sum of its digits i.e. 1 + 3 + 2 = 6 is exactly divisible by 3. But 275 is not divisible by 3 as sum of the digits i.e. 2 + 7 + 5 = 14 is not divisible by 3.

**4:** If last two digits of the number are either 0s or multiple of 4.

Example: 24, 64, 900 and 9196 are divisible by 4. But 18, 22 and 994 are not divisible by 4.

Note: A number having odd number at unit's place is never divisible by 2 or by any other even number.

**5:** If unit digit of the number is 0 or 5.

Example: 135, 75290 are divisible by 5. But 1019, 50852 are not divisible by 5.

**8:** If its last three digits are either 0s or a multiple of 8. Example: 721000, 5144 are divisible by 8. But 2433 and 8996 are not divisible by 8.

**9:** If sum of digits of the number is exactly divisible by 9.

Note: While totaling the digits, any digit or pair of digits totaling to 9 may be ignored and the final sum is reduced to one digit.

Example: 216 is divisible by 9 as sum of digits i.e. 2 + 1 + 6 = 9 is divisible by 9. 3294 is divisible by 9 as sum of digits is divisible by 9.

Note: If we ignore both 9 while totaling the digits of 3294, then we get 0 as the sum of digits. If we strike out all 9 while totaling the digits of a number, sum of digits is always 0 when the number is exactly divisible by 9.

**10:** If unit's digit of the number is 0. Example: 2760, 38610 are divisible by 10. But 1531, 2376 are not divisible by 10.

**11:** If difference between sum of digits at the even places and sum of digits at odd places is either 0 or a multiple of 11.

Example1: 3256: Difference between

(3 + 5) and (2 + 6) is 0. The number is divisible by 11.

Example2: 2948: Difference between

(2 + 4) and (9 + 8) is 11, which is a

multiple of 11. The number is divisible by 11.

**25:** If last two digits of the number are either 0s or divisible by 25.

Numbers divisible by 25 must end in 00, 25, 50 or 75.

Example: 2125, 4350, 1375, 7800 are

divisible by 25. But 165 , 4327are not

divisible by 25.

Note: The methods explained above can be used to find whether a divisor exactly divides a given number if the divisor can be written as a product of two or more co-prime numbers for which Divisibility rules have been explained above.

Co-prime numbers are relatively prime to each other. They are also called as relatively prime numbers.

**6:** If the number is divisible by 2 and 3 (Co-prime numbers)

Examples: 1. Is 204 divisible by 6?

204 is divisible by 2. 204 is divisible by 3. 204 is divisible by 6.

2. Is 1431 divisible by 6?

1431 is not divisible by 2. The number is not divisible by 6.

3. Is 2156 divisible by 6?

2156 is divisible by 2. 2156 is not divisible by 3. The number is not divisible by 6.

In the above examples, we see that a number is divisible by the given divisor, if all the co-prime factors of the given divisor divide the given number exactly.

**12:** If the number is divisible by 3 and 4 (Co-prime numbers)

Example: Is 1068 divisible by 12?

1068 is divisible by 3. 1068 is divisible by 4. 1068 is divisible by 12.

**14:** If the number is divisible by 2 and 7 (Co-prime numbers)

Example: Is 938 divisible by 14?

938 is divisible by 2. 938 is divisible by 7. 938 is divisible by 14.

**15:** If the number is divisible by 3 and 5 (Co-prime numbers)

Example: Is 345 divisible by 15?

345 is divisible by 3. 345 is divisible by 5. 345 is divisible by 15.

**18:** If the number is divisible by 2 and 9 (Co-prime numbers)

Example: Is 1134 divisible by 18?

1134 is divisible by 2. 1134 is divisible by 9. 1134 is divisible by 18.

**REMAINDERS CONCEPT**

If a number is exactly divisible by a divisor, remainder is '0' and in the other cases remainder may be any number between one and one less than the divisor. Generally, remainders are obtained by actually dividing the given number by the divisor. Using the Rules of Divisibility Test, we can also find the remainder without actually dividing the number.

**RULES**

1. A certain number when successively divided by two different numbers leaves some remainders. The same number when divided by the product of the two divisors will leave remainder equal to: First Divisor x Second Remainder + First Remainder.

2. Two numbers 'A' and 'B’ when separately divided by X, leaves remainders ‘a’ and ‘b’ respectively.(A B) when divided by X, will leave remainder (a b).

Logic:

(X + a) (X + b) = X2 + (a + b) X + (ab)

**L.C.M & H.C.F**

LCM i.e. least common multiple is a number which is multiple of two or more than two numbers.

For example: The common multiples of 18 and 24 are 72,144 and so on. Therefore, L.C.M is smallest positive number that is multiple of both. Here, L.C.M is 72.

HCF i.e. highest common factor are those integral values of number that can divide that number.

For example: The common factors of 18 and 24 are 1, 2, 3, and 6. Therefore, H.C.F is the greatest positive number that is factor of both. Here, H.C.F is 6.

**Properties of L.C.M. and H.C.F. :**

Product of two numbers (a, b) = H.C.F (a, b) L.C.M (a, b)   
  
H.C.F. of given numbers always divides their L.C.M.

**Note:**

Largest number which divides x, y, z to leave same remainder

= H.C.F. (y-x, z-y, x-z)

Largest number which divides x, y, z to leave remainders a, b, c respectively

= H.C.F. (x-a, y-b, z-c)   
  
Least number which when divided by x, y, z and leaves a remainder R in each case

=L.C.M. (x, y, z) + R

**'BODMAS’ rule:**

This rule depicts the correct sequence in which the operations are to be executed, so as to find out the value of a given expression.

Here, “B” stands for 'Brackets’

“O" for 'of',

"D" for 'Division,

“M” for Multiplication

“A” for 'Addition',

“S” for 'Subtraction'.

**‘VBODMAS’**

1. The first step is to evaluate the expression under vinculum.
2. The next step is to evaluate all the expressions in the brackets.
3. After removing the brackets, we must use the following operations strictly in the following order:
4. Of, Orders (i.e. Powers and Square Roots, etc.)
5. Division, Multiplication
6. Addition, Subtraction

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| 1.3 Quiz |

**Q1.** What are the last two digits of 72008? (a) 21 (b) 61 (c) 01 (d) 41 (e) 81

**Q2.** Consider all four digit numbers for which the first two digits are equal and the last two digits are also equal. How many such numbers are perfect squares?

(a) 3 (b) 2

(c) 4 (d) 0

(e) 1

**Q3.** Find the number of zeroes in 48! (a) 9 (b) 12 (c) 11 (d) 10 (e) 0

**Q4.** The rightmost non-zero digit of the number 302720 is:

(a) 1 (b) 3 (c) 7 (d) 9

**Q5.** A shop stores *x* kg of rice. The first customer buys half this amount plus half a kg of rice. The second customer buys half the remaining amount plus half a kg of rice. Then the third customer also buys half the remaining amount plus half a kg of rice. Thereafter, no rice is left in the shop. Which of the following best describes the value of *x*?

(a) 2 ≤ *x* ≤ 6 (b) 5 ≤ *x* ≤ 8 (c) 9 ≤ *x* ≤ 12 (d) 11≤ *x* ≤ 14 (e) 13 ≤ *x* ≤ 18

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| 1.4 Worked out examples |

**Q1.** A certain number when successively divided by 4 and 7 leaves the remainders 2 and 3 respectively. What is the remainder if the same number is divided by 28?

(a) 5

(b) 14

(c) 17

(d) 26

**Explanatory Answer:** Remainder = First Divisor x Second Remainder + First Remainder

= 4 x 3 + 2 = 14

**Q2.** Find the remainder on dividing 25 x 26 by 24?

**Explanatory Answer:**

25 and 26 when divided by 24, leave remainders 1 and 2 respectively.

25 x 26 when divided by 24, leaves remainder 1 x 2 = 2

**Q3.** Find the remainder on dividing 17 x 27 x 95 x 36 by 8.

**Explanatory Answer:**

17, 27, 95 and 36 when divided by 8, leaves remainders of 1, 3, 7 and 4 respectively.

17 x 27 x 95 x 36 when divided by 8, leaves remainder 1x3 x7 x4= 84. Remainder cannot exceed the divisor. Remainder is 4 i.e. remainder obtained on dividing 84 by 8.

**Q4.** What value must be given to \* to make 8597\*65 exactly divisible by 11?

(a) 0

(b) 2

(c) 4

(d) 7

**Explanatory Answer:**

Sum of digits at odd places (from left side) = 8 + 9 + \* + 5 = 22 + \*

Sum of digits at even places (from left side) = 5 + 7 + 6 = 18

Difference between the two sums = 4 + \*

We know that any number is divisible by 11, if the difference is ‘0' or a multiple of 11. The difference between 22 + \* and 18 will be 11 when \* = 7

**Q5.** Three bells toll at the interval of 12, 15 and 18 minutes respectively. If all the three bells toll together at 8 a.m., when will they toll together again?

(a) 9 a.m.

(b) 10 a.m.

(c) 11 a.m.

(d) 1 p.m.

**Explanatory Answer:**

LCM of 12, 15 and 18 is 3 x 4 x 5 x 3 = 180 minutes = 3 hours. The bells will toll together 3 hours after 8 a.m. i.e. at 11 a.m.

**Q6.** Three persons A, B and C can complete one round of a circular track in 6, 15, and 20 minutes respectively. If they start together at 9 a.m. from a certain point, at what time will they be together again at the starting point?

(a) 10 a.m.

(b) 10.30 a.m.

(c) 11 a.m.

(d) 11.30 a.m.

**Explanatory Answer:**

LCM of 6, 15 and 20 is 2 x 3 x 5 x 2 = 60 minutes =1 hour. They will be again at starting point 1 hr after 9 a.m. i.e.10 a.m.

**Q7.** Least number which when divided by 35, 45, 55 and leaves remainder 18, 28 and 38 is:

**Explanatory Answer:**

In this case we will evaluate L.C.M.  
Here the difference between every divisor and remainder is same i.e. 17.Therefore, required number

= L.C.M. of (35, 45, 55) - 17

= (3465-17)

= 3448.

**Q8.** What is the unit digit of 4925?

41 is 4.

42 is 16.

43 is 64

44 is 256.

The power pattern for 4 is 4 & 6 with 2 cycle patterns for the last digit.

Last digit of 4odd is 4 & 4even is 6

Last digit of 4925 is 4.

**Q9.** Greater number of 4 digits which is divisible by each one of 12, 18, 21 and 28 is:

**Explanatory Answer:**

L.C.M. of 12,18,21,28 = 252  
therefore, required number must be divisible by 252.  
Greatest four digit number = 9999

On dividing 9999 by 252, remainder = 171  
Therefore, 9999-171 = 9828.

**Q10.** Find the number of zeroes in 6!

**Explanatory Answer:**

Zeroes are formed by a combination of 2 x 5.

Hence, the number of zeroes will depend on the number of pairs of 2s and 5s that can be formed.

6! = 6 x 5x 4 x 3 x 2x 1

= (3x 2)x(5) x(2x 2)x(3)x(2)x(1).

The above expression will have only one pair of 5 x 2, since there is only one 5 and an abundance of 2s. So only one zero at the end in 6!

**Note:** It is clear that in any factorial value, the number of 5s will always be lesser than the number of 2s. Hence, all we need todo is to count the number of 5s.

**Q11.** 45 - 4 x 6 - 5 + 14 / 7 **=?**

**Explanatory Answer:**

45 - 4 x 6 - 5 + 14 / 7

= 45 - 24 - 5 + 2

= 18

**Q12.** 21 / 3 (10 - 3) - 20 + 1 =?

**Explanatory Answer:**

21 / 3 (10 – 3) - 20 + 1

=21 / 3 x 7 - 20 + 1

=7 x 7 - 20 + 1

=49 - 20 + 1

= 30

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| 1.5 Class Work Problems |

**Q1.** Let x, y and z be distinct integers, x and y are odd and positive, and z is even and positive. Which one of the following statements cannot be true? (a) (x – z)2 y is even (b) (x – y)y2 is odd (c) (x – z)y is odd (d) (x – y)2 z is even

**Q2.** All the page numbers from a book are added, beginning at page 1. However, one page number was mistakenly added twice. The sum obtained was 1000. Which page number was added twice? (a) 44 (b) 45 (c) 10 (d) 12

**Q3.** Raju has 128 boxes with him. He has to put at least 120 oranges in one box and 144 at the most. Find the least number of boxes which will have the same number of oranges. (a) 5 (b) 103 (c) 6 (d) cannot be determined

**Q4.** If x > 5 and y < – 1, then which of the following statements is true? (a) (x + 4y) > 1 (b) x > – 4y (c) – 4x < 5y (d) None of these

**Q5.** In a 4-digit number, the sum of the first two digits is equal to that of the last two digits. The sum of the first and last digits is equal to the third digit. Finally, the sum of the second and fourth digits is twice the sum of the other two digits. What is the third digit of the number? (a) 5 (b) 8 (c) 1 (d) 4

**Q6.** A red light flashes 3 times per minute and a green light flashes 5 times in two minutes at regular intervals. If both lights start flashing at the same time, how many times do they flash together in each hour? (a) 30 (b) 24 (c) 20 (d) 60

**Q7.** Ashish is given Rs 158 in one rupee denomination. He has been asked to allocate them into a number of bags such that any amount required between Re 1 and Rs 158 can be given by handing out a certain number of bags without opening them. What is the minimum number of bags required? (a) 11 (b) 12 (c) 13 (d) None of these

**Q8.** Let b be a positive integer and a = b2 – b. If b ≥4, then a2 – 2a is divisible by: (a) 15 (b) 20 (c) 24 (d) None of these

**Q9.** A change making machine contains 1 rupee, 2 rupee and 5 rupee coins. The total number of coins is 300. The amount is Rs 960. If the number of 1 rupee coins and the number of 2 rupee coins are interchanged, the value comes down by Rs 40. The total number of 5 rupee coins is: (a) 100 (b) 14 (c) 60 (d) 150

**Q10.** What is the least number which when doubled will be exactly divisible by 12, 14, 18 and 22? (a) 1286 (b) 1 (c) 1216 (d) 1386

**Q11.** Two boys are playing on a ground. Both the boys are below 10 years. Age of the younger boy is equal to the cube root of the product of the age of the two boys. If we place the digit representing the age of the younger boy to the left of the digit representing the age of the elder boy, we get the age of the father of the younger boy. Similarly, we place the digit representing the age of the elder boy to the left of the digit representing the age of the younger boy and divide the figure by 2, we get the age of the mother of the younger boy. The mother of the younger boy is younger than his father by 3 years. Then, what is the age of the younger boy? (a) 3 (b) 2 (c) 4 (d) None of these

**Q12.** What is the least number which when divided by 5, 6, 7 and 8 leaves a remainder 3, but when divided by 9 leaves no remainder?

(a) 1108 (b) 1683 (c) 2007 (d) 3363

**Q13.** The H.C.F. of two numbers is 5 and their L.C.M. is 150. If one of the numbers is 25, then the other is: (a) 30 (b) 28 (c) 24 (d) 20

**Q14.** Which of the following integers has the most number of divisors?

(a) 101 (b) 99 (c) 182 (d) 176

**Q15.** The least number which should be added to 28523 so that the sum is exactly divisible by 3, 5, 7 and 8 is:

(a) 41 (b) 42 (c) 32 (d) 37

**Directions for Questions (Q16-Q21):** Evaluate the following expressions.

**Q16.** 44 / 22× 6 =

**Q17.** 44 / (22×6) =

**Q18.** {20 + } / (2 + 3) =

**Q19.** 20 + / (2 + 3) =

**Q20.** 20 + / 2 + 3 =

**Q21.** 1 + [2 – {3 + (4 – 5 + 6)}]

**Q22.** What is the unit digit in 7 105? (a) 1 (b) 7 (c) 9 (d) 5

**Q23.** = ?

(a) 144 (b) 864 (c) 2 (d) 4

**Q24.** A number when divided successively by 4 and 5 leaves remainders 1 & 4 respectively. When it is successively divided by 5 and 4, then the respective remainders will be

(a) 1,2 (b) 2, 3 (c) 3, 2 (d) 4, 1

**Q25.** A 3 digit number 4a3 is added to another 3 digit number 984 to give a 4 digit number 13b7, which is divisible by 11. Then (a + b) =? (a) 10 (b) 11 (c) 12 (d) 15

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| 1.6 Home assignment |

**Directions for Questions (Q1-Q4):** Answer the questions on the basis of the information given below.

A country has the following types of traffic signals.

3 red lights = stop;

2 red lights = turn left;

1 red light = turn right;

3 green lights = go at 100 kmph speed;

2 green lights = go at 40 kmph speed;

1 green light = go at 20 kmph speed.

A motorist starts at a point on a road and follows all traffic signals literally. His car is heading towards the north. He encounters the following signals (the time mentioned in each case below is applicable after crossing the previous signal).

Starting Point – 1 green light;

After half an hour, 1st signal – 2 red & 2 green lights;

After 15 minutes, 2nd signal – 1 red light;

After half an hour, 3rd signal – 1 red & 3 green lights;

After 24 minutes, 4th signal – 2 red & 2 green lights;

After 15 minutes, 5th signal – 3 red lights;

**Q1.** The total distance traveled by the motorist from the starting point till the last signal is (a) 90 km (b) 100 km (c) 120 km (d) None of these

**Q2.** What is the position (radial distance) of the motorist when he reaches the last signal: (a) 45 km directly north of starting point. (b) 30 km directly to the east of the starting point. (c) 50 km. away to the northeast of the starting point. (d) 45 km away to the northwest of the starting point.

**Q3.** After the starting point if the 1st signal were 1 red and 2 green lights, what would be the final position of the motorist: (a) 30 km to the west and 20 km to the south. (b) 30 km to the west and 40 km to the north. (c) 50 km to the east and 40 km to the north. (d) Directly 30 km to the east.

**Q4.** If at the starting point, the car was heading towards south, what would be the final position motorist: (a) 30 km to the east and 40 km to the south. (b) 50 km to the east and 40 km to the south. (c) 30 km to the west and 40 km to the south. (d) 50 km to the west and 20 km to the north.

**Q5.** 76n – 66n, where n is an integer > 0, is divisible by (a) 13 (b) 127 (c) 559 (d) All of these

**Q6.** What is the greatest possible length which can be used to measure exactly the lengths 8 m, 4 m 20 cm and 12 m 20 cm? (a) 10 cm (b) 30 cm (c) 25 cm (d) 20 cm

**Q7.** The product of two numbers is 2028 and their HCF is 13. How many numbers of such pairs are possible? (a) 4 (b) 3 (c) 2 (d) 1

**Q8.** N is the greatest number which divides 1305, 4665 and 6905 and gives the same remainder in each case. What is the sum of the digits in N? (a) 4 (b) 3 (c) 6 (d) 5

**Q9**. Suppose n is an integer such that the sum of digits of n is 2, and 1010 ≥n ≥1011. The number of different values of n is:

(a) 11 (b) 10 (c) 9 (d) 8

**Q10.** After the division of a number successively by 3, 4 and 7, the remainders obtained are 2, 1 and 4 respectively. What will be the remainder if 84 divide the same number? (a) 80 (b) 76 (c) 41 (d) 53

**Q11.** When 2256 is divided by 17 the remainder would be

(a) 14 (b) 12 (c) 16 (d) None of these

**Q12.** Three pieces of cakes of weight lbs, 6 lbs and 7 lbs respectively are to be divided into parts of equal weights. Further, each part must be as heavy as possible. If one such part is served to each guest, then what is the maximum number of guests that could be entertained? (a) 54 (b) 72 (c) 20 (d) None of these

**Q13.** At a bookstore, “MODERN BOOK STORE” is flashed using neon lights. The words are individually flashed at intervals of 2 , 4 , 5 seconds respectively and each word is put off after a second. The least time after which the full name of the bookstore can be read again is: (a) 49.5 seconds (b) 73.5 seconds (c) 1744.5 seconds (d) 855 seconds

**Q14.** A child was asked to add first few natural numbers (that is, 1 + 2 + 3 …) so long his patience permitted. As he stopped, he gave the sum as 575. When the teacher declared the result wrong the child discovered he had missed one number in the sequence during addition. The number he missed was:

(a) less than 10 (b) 10 (c) 15 (d) more than 15

**Q15.** A rich merchant had collected many gold coins. He did not want anybody to know about them. One day, his wife asked, “How many gold coins do we have?” After pausing a moment, he replied, “Well! If I divide the coins into two unequal numbers, then 48 times the difference between the two numbers equals the difference between the squares of the two numbers.” The wife looked puzzled. Can you help the merchant’s wife by finding out how many coins the merchant has?

(a) 53 (b) 96 (c) 43 (d) None of these

**Q16.** Six bells start ringing together and ring at intervals of 4, 8, 10, 12, 15 and 20 seconds respectively. How many times will they ring together in 60 minutes? (a) 31 (b) 15 (c) 16 (d) 30

**Q17.** Three numbers which are co-prime to each other are such that the product of the first two is 119 and that of the last two is 391. What is the sum of the three numbers? (a) 47 (b) 43 (c) 53 (d) 51

**Q18.** What is the greatest number which divides 24, 28 and 34 and leaves the same remainder in each case?

(a) 1 (b) 2 (c) 3 (d) 4

**Q19.** What is the least number which when divided by 8, 12, 15 and 20 leaves in each case a remainder of 5? (a) 125 (b) 117 (c) 132 (d) 112

**Q20.** What is the smallest number which when diminished by 12, is divisible 8, 12, 22 and 24? (a) 276 (b) 264 (c) 272 (d) 268

**Directions for Questions (Q21-Q25):** Evaluate the following expressions.

**Q21.** 24 ÷ (4 + 2) =

**Q22.** 12 x (8 - 7) =

**Q23.** (4 x 8) /0.5 =

**Q24.** 72 / (3 + 15) =

**Q25.** 18 / 9 - 2 =

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| 1.7 Self Assessment |

**Q1.** Three travelers are sitting around a fire, and are about to eat a meal. One of them has five small loaves of bread; the second has three small loaves of bread. The third has no food, but has eight coins. He offers to pay for some bread. They agree to share the eight loaves equally among the three travelers, and the third traveler will pay eight coins for his share of the eight loaves. All loaves were the same size. The second traveler (who had three loaves) suggests that he be paid three coins and that the first traveler be paid five coins. The first traveler says that he should get more than five coins. How much the first traveler should get? (a) 5 (b) 7 (c) 1 (d) None of these

**Q2.** A piece of string is 40 centimeters long. It is cut into three pieces. The longest piece is 3 times as long as the middle-sized piece and the shortest piece is 23 centimeters shorter than the longest piece. Find the length of the shortest piece. (a) 27 (b) 5 (c) 4 (d) 9

**Q3.** The owner of a local jewellers store hired 3 watchmen to guard his diamonds, but a thief still got in and stole some diamonds. On the way out, the thief met each watchman, one at a time. To each he gave ½ of the diamonds he had then, and 2 more besides. He escaped with one diamond. How many did he steal originally? (a) 25 (b) 36 (c) 40 (d) None of these

**Q4.** If x = (163 + 173 + 183 + 193), then x divided by 70 leaves a remainder of:

(a) 0 (b) 1 (c) 69 (d) 35

**Q5.** Let n! = 1 × 2 × 3 × ... × n for integer n ≥1. If p = 1! + (2 × 2!) + (3 × 3!) + ... + (10 × 10!), then p + 2 when divided by 11! leaves a remainder of:

(a) 10 (b) 7 (c) 0 (d) 1

**Q6.** The digits of a three-digit number A are written in the reverse order to form another three-digit number B. If B > A and B – A is perfectly divisible by 7, then which of the following is necessarily true?

(a) 100 < A < 299 (b) 106 < A < 305

(c) 112 < A < 311 (d) 118 < A < 317

**Q7.** Let S be the set of five-digit numbers formed by the digits 1, 2, 3, 4 and 5, using each digit exactly once such that exactly two odd positions are occupied by odd digits. What is the sum of the digits in the rightmost position of the numbers in S?

(a) 228 (b) 216

(c) 294 (d) 192

**Q8.** The integers 1, 2… 40 are written on a blackboard. The following operation is then repeated 39 times: In each repetition, any two numbers, say ‘a’ and ‘b’, currently on the blackboard are erased and a new number ‘a + b – 1’ is written. What will be the number left on the board at the end? (a) 820 (b)821 (c) 781 (d)819 (e) 780

**Q9.** Suppose, the seed of any positive integer *n* is defined as follows: seed(*n*) = *n,* if *n* < 10 = seed(*s*(*n*)), otherwise, where *s*(*n*) indicates the sum of digits of *n.*

For example, seed (7) = 7, seed (248) = seed (2 + 4 + 8) = seed (14) = seed (1 + 4) = seed (5) = 5.

How many positive integers *n*, such that

*n* < 500, will have seed (*n*) = 9? (a) 39 (b) 72 (c) 81 (d) 108 (e) 55

**Q10.** A boy divided the numbers 7654, 8506 and 9997 by a certain largest number and he gets same remainder in each case. What is the common remainder? (a) 156 (b) 199 (c) 211 (d) 231

**Q11.** What is the greatest number which on dividing 1223 and 2351 leaves remainders 90 and 85 respectively? (a) 1133 (b) 127 (c) 42 (d) 1100

**Directions for Questions (Q12-Q13):** Answer the questions on the basis of the information given below.

A, B, C, D, E and F are six positive integers such that

B + C + D + E = 4A C + F = 3A C + D + E = 2F F = 2D E + F = 2C + 1

If A is a prime number between 12 and 20, then

**Q12.** Which of the following must be true? (a) D is the lowest integer & D= 14 (b) C is the greatest integer and C=23 (c) B is the lowest integer & B=12 (d) F is the greatest integer and F=24 (e) A is the lowest integer & A=13

**Q13.** The value of F is (a) 14 (b) 16 (c) 20 (d) 24 (e) 28

**Q14.** A, B and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and c in 198 seconds, all starting at the same point. After what time will they again at the starting point? (a) 36 minutes 22 seconds (b) 46 minutes 22 seconds (c) 36 minutes 12 seconds (d) 46 minutes 12 seconds

**Q15.** There are 200 questions in a 3 hr examination. Among these questions are 50 mathematics problems. It is suggested that twice as much time be spent on each mathematics problem as for each other question. How many minutes should be spent on mathematics problems (a) 36 (b) 72 (c) 64 (d) 60 (e) 100

**Q16.** What is the least multiple of 7 which leaves a remainder of 4 when divided by 6, 9, 15 and 18? (a) 364 (b) 350 (c) 343 (d) 371

**Directions for Questions (Q17–Q20):** Evaluate the following expressions.

**Q17.** 10+ 162 / 4=

**Q18.** (10+ 162) / 4=

**Q19.** 9 /( ( 5 - 2) ×5) =

**Q20.** 9 / ( 5 - 2) × 5 =

**Chapter 2**

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| --- |
| Set Theory |

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| 2.1 Learning objective: |

To learn about the basics of Sets and its applications.

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| 2.2 Introduction: |

A set is any unordered collection of distinct objects. These objects are called the elements or members of the set.

* Objects, elements and members of a set are synonymous terms.
* Sets are usually denoted by capital letters A, B, C, X, Y, Z, etc.
* The elements of a set are represented by small letters a, b, c, x, y, z, etc.

There are two methods of representing a set:

**Roster or tabular form**

All the elements of a set are listed, the elements are being separated by commas and are enclosed within braces { }.

For example:

The set of all even positive integers less than 7 is described in roster form as {2, 4, 6}.

The set of all vowels in the English alphabet is {a, e, i, o, u}

The set of all natural numbers which divide 42 is {1, 2, 3, 6, 7, 14, 21, 42}

**Set-builder form**

All the elements of a set possess a single common property which is not possessed by any element outside the set. For example, in the set {a, e, i, o, u}, all the elements possess a common property, namely, each of them is a vowel in the English alphabet, and no other letter possess this property. Denoting this set by V, we write

**V = {x : x is a vowel in English alphabet}**

A = {x : x is a natural number and 3 < x < 10} is read as "the set of all x such that x is a natural number and x lies between 3 and 10. Hence, the numbers 4, 5, 6, 7, 8 and 9 are the elements of the set A.

**Finite Set:**

A set 'A' is said to be finite if it is either an empty set or contains finite number of elements

**Example:**

The Set of vowels in English alphabet is finite.

**Infinite Set:**

A Set, which is not finite. That is the elements which are too innumerable to count are called infinite set.Let A be the set of points on a line. Then A is infinite.

**Empty Set:**

A set which does not contain any element is called the empty set or the null set or the void set and is denoted by 𝝓

Let A = {x : 1 < x < 2, x is a natural number}. Then A is the empty set, because there is no natural number between 1 and 2

**Equal Sets:**

Two sets A and B are said to be equal if they have exactly the same elements and we write A = B. Otherwise, the sets are said to be unequal and we write A≠B

A be the Set of prime numbers less than 6 and P the set of prime factors of 30. Then A and P are equal, since 2, 3 and 5 are the only prime factors of 30 and also these are less than 6.

**Subset:**

A set A is said to be a subset of a set B if every element of A is also an element of B.

In other words, A ⊂ B if whenever *a 𝜖* A, then *a 𝜖* B. It is often convenient to use the symbol “⇒” which means implies. Using this symbol, we can write the definition of subset as follows A ⊂ B if *a 𝜖* A ⇒ *a 𝜖* B

**The set Q of rational numbers is a subset of the set R of real numbers, and we write Q ⊂ R**.

If A is a subset of B we say that B contains A or B is a superset of A and is denoted by B ⊃ A. If A is not a subset of B then this is denoted by A ⊂B.

**Every set is a subset of itself.**

**Empty set is a subset of every set.**

**If A is a finite set of cardinality n, then total number of subsets of A are 2n.**

**Power Set:**

If A is any set, then the set of all subsets of A is called the power set of A and is denotes by P(A).

**Example:** If A = {1, 2, 3}, then P(A) = {𝝓,{1}, {2}, {3}, {1,2}, {1,3), {2, 3}, {1,2,3}}

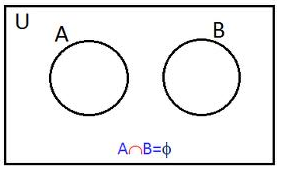
**Universal Set:**

A universal set is the collection of all objects in a particular context or theory. All other sets in that framework constitute subsets of the universal set, which is denoted as an uppercase italic letter *U*. The objects themselves are known as elements or members of *U*.

**Example:**

The universal set is the set of real numbers, while considering the set of Natural numbers, Whole numbers, Integers and Rational numbers.

**Disjoint Sets:**

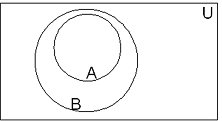
****If A and B are such that they have no common elements that is A ∩ B is 𝝓 then Set A and Set B are called disjoint or mutually exclusive sets.

**Venn Diagrams:**

A Venn diagram is a pictorial representation of sets, wherein they are shown as enclosed areas. Typically, the universal set *U* is represented by the area within a rectangle, and the other sets as circles placed within the rectangle.

**Example:**

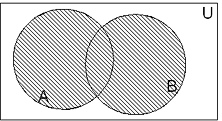
A as subset of B: A⊆B Can be represented as



A⊂ B

**Basic Set Operations:**

**Union of sets:**

 AUB

The union of two sets A and B is the set C which consists of all those elements which are either in A or in B (including those which are in both). In symbols, we write.

**A ∪ B = { x : x ∈A or x ∈B }**

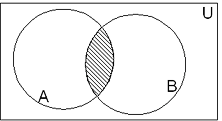
**Some properties of the operation of Union:**

* A ∪ B = B ∪ A (Commutative law)
* (A ∪ B ) ∪ C = A ∪ ( B ∪ C) (Associative law )
* A ∪ 𝝓 = A (Law of identity element, 𝝓 is the identity of *∪*)
* A ∪ A = A (Idempotent law)
* *U* ∪ A = *U*  (Law of *U*)
* A ∪ A′=*U*

**Intersection of sets:**

The intersection of sets A and B is the set of all elements Which are common to both A and B. The symbol ‘∩’ is used to denote the intersection.

**A ∩ B={x: x∈ A and x ∈B}**



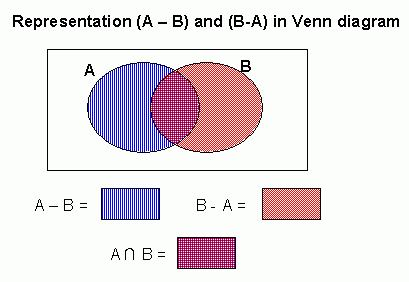
**Some properties of operation of Intersection:**

* A ∩ B = B ∩ A (Commutative law).
* (A ∩ B) ∩ C = A ∩ (B ∩ C) (Associative law).
* 𝝓 ∩ A = 𝝓, U ∩ A = A (Law of φ and *U*)
* A ∩ A = A (Idempotent law)
* A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) (Distributive law)

**Difference of sets:**

The difference of the sets A and B in this order is the set of elements which belong to A but not to B. Symbolically, we write

A – B and read as “ A minus B”.



A = { 1, 2, 3, 4, 5, 6}, B = { 2, 4, 6, 8 }.Then A – B = { 1, 3, 5 } and B – A = { 8 }

**Some properties of the operation of difference:**

* A – B ≠ B – A
* Sets A-B, A∩B and B-A are mutually disjoint sets
* A⊂ B then A-B= 𝝓
* A-B ⊂ A

**Complement of a Set:**

Let *U* be the universal set and Set A be the subset of *U*. Complement of a set A with respect to the universal set *U* is the set of all those elements of *U* which are not elements of A and is denoted by A′ or Ac

**A′ = {x : x ∈ U and x ∉ A }**

If *U* = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and

A = {1, 3, 5, 7, 9} then A′= { 2, 4, 6, 8,10 }

**Some Properties of Complement Sets:**

* Complement laws:

(i) A ∪ A′ = *U*

(ii) A ∩ A′ = 𝝓

* De Morgan’s law:

(i) (A ∪ B)′ = A′ ∩ B′

(ii) (A ∩ B )′ = A′ ∪ B′

* Law of double complementation:

(A′) ′ = A

**Symmetric Difference of Two Sets:**

Let A and B are two Sets. Then Symmetric Difference of Two Sets A and B is the set

(A-B)∪ (B-A) and is denoted by A △ B

Example:

A= {1, 2, 3, 4} B= {3, 4, 5, 6} Then

A △ B= {1, 2, 5, 6}

**Cardinality of a Set:**

The number of distinct elements in a set is called the cardinality (or order) of the set. If a finite set A has n distinct elements, the cardinality of the set is n and is denoted by 0(A) or n (A). The cardinality of empty set is 0.

Example: Cardinality of A = {x, y, z, t} is 4.

**Principles of Inclusion and Exclusion:**

Let A and B be finite sets

n(A ∪ B ) = n ( A ) + n ( B )-n(A∩ B**)**

If A ∩ B = 𝝓, then, n(A ∪ B ) = n ( A ) + n ( B )

n(A ∪ B ∪ C)= n (A) + n ( B ) + n ( C ) – n ( A ∩ B ) – n ( B ∩ C)– n ( A ∩ C ) +n ( A ∩ B ∩ C )

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| 2.3 Quiz |

**Q1.** Imagine that this Hotel-infinity has infinite number of rooms numbered 1, 2, 3... Now all rooms are full and a man comes and asks the manager for a room. Manager replies all rooms are full, but still I will accommodate for you one. How does he do?

**Q2**. Imagine that this Hotel-infinity has infinite number of rooms numbered 1, 2, 3... Now all rooms are full and an infinite number of guests comes and ask the manager for a room. Manager replies all rooms are full, but still I will accommodate for you all. How does he do?

**Q3.** Let *U* = {1, 2, 3, 4, 5, 6, 7, 8, 9},

A = {1, 2, 3, 4} and B = {2, 4, 6, 8}.

(i) Find A′

(ii) Find B′

**Q4.** If n (A - B) = 18, n (A ∪ B) = 70 and

n (A ∩ B) = 25, then find n(B)?

**Q5.** In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both coffee and tea?

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| 2.4 Worked out examples |

**Directions for questions (Q1–Q3):** If X = {g, o, a, l, s} then how many subsets of X

**Q1**. Contain "a" but not "s"?

(a) 16

(b) 8

(c) 32

(d) 64

**Explanatory Answer:**

The subsets must contain “a” but not “s”. The total elements excluding these two is three. So total number of subsets is 23=8.

**Q2.** Contain exactly 3 elements?

(a) 10

(b) 8

(c) 15

(d) 32

The correct choice is (a) 10 [Hints: ]

**Q3.** A bag contains 100 apples, 100 oranges, 100 bananas and 100 pears. Every minute you choose one fruit from the bag. How long will it take to ensure that you have at least a dozen fruit of the same kind?

**Explanatory Answer:**

Suppose we have already chosen 11 apples, 11 oranges, 11 bananas and 11 pears. The next choice will ensure that we have a dozen of one type of fruit. Therefore it takes 4(11) +1 = 45 minutes to ensure that we have at least on dozen of one type of fruit

**Q4.** In a group of 60 people, 27 like coffee and 42 like tea and each person likes at least one of the two. How many like both coffee and tea?

**Explanatory Answer:**

Let A = Set of people who like coffee.   
    B = Set of people who like tea.   
Given   
n (A ∪ B) = 60            n (A) = 27

n (B) = 42 then,

n(A ∩ B) = n(A) + n(B) - n(A ∪ B)   
           = 27 + 42 - 60   
            = 69 - 60 = 9   
Therefore, 9 people like both tea and coffee.

**Q5.** In a competition, a school awarded medals in different categories. 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many received medals in exactly two of these categories?

**Explanatory Answer:**

Let A = set of persons who got medals in dance.   
B = set of persons who got medals in dramatics.   
C = set of persons who got medals in music.

Given, n(A) = 36, n(B) = 12, n(C) = 18

n(A ∪ B ∪ C) = 45, n(A ∩ B ∩ C) = 4

We know that number of elements belonging to exactly two of the three sets A, B, C

= n(A ∩ B) + n(B ∩ C) + n(A ∩ C)

– 3 n(A ∩ B ∩ C)   
= n(A ∩ B) + n(B ∩ C) + n(A ∩ C) - 3 × 4       ……..(i)   
n(A ∪ B ∪ C) = n(A) + n(B) + n(C) - n(A ∩ B) - n(B ∩ C) - n(A ∩ C) + n(A ∩ B ∩ C)   
Therefore, n(A ∩ B) + n(B ∩ C) + n(A ∩ C) = n(A) + n(B) + n(C) + n(A ∩ B ∩ C) - n(A ∪ B ∪ C)   
From (i) required number   
= n(A) + n(B) + n(C) + n(A ∩ B ∩ C) - n(A ∪ B ∪ C) - 12   
= 36 + 12 + 18 + 4 - 45 - 12   
= 70 - 67   
= 3

**Q6.** Each student in a class of 40 plays at least one indoor game Chess, Carom and Scrabble. 18 play chess, 20 play scrabble and 27 play carom. 7 play chess and scrabble, 12 play scrabble and carom and 4 play Chess, Carom and Scrabble. Find the number of students who play

(i) Chess and Carom.

(ii) Chess, Carom but not Scrabble.

**Explanatory Answer:**

Let A be the set of students who play Chess   
B be the set of students who play Scrabble   
C be the set of students who play Carom   
Therefore, We are given n(A ∪ B ∪ C) = 40,

n(A) = 18,         n(B) = 20,         n(C) = 27,   
n(A ∩ B) = 7, n(C ∩ B) = 12,

n(A ∩ B ∩ C) = 4   
We have,  
n(A ∪ B ∪ C) = n(A) + n(B) + n(C) - n(A ∩ B) - n(B ∩ C) - n(C ∩ A) + n(A ∩ B ∩ C)   
Therefore,

40 = 18 + 20 + 27 - 7 - 12 - n(C ∩ A) + 4   
40 = 69 – 19 - n(C ∩ A)   
40 = 50 - n(C ∩ A) n(C ∩ A) = 50 - 40   
n(C ∩ A) = 10

Therefore, Number of students who play Chess and carom are 10.   
Also, number of students who play Chess, Carom and not Scrabble.   
= n(C ∩ A) - n(A ∩ B ∩ C)   
= 10 – 4 = 6

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| 2.5 Class Work Problems |

**Q1**. The number of distinct elements in the set (1, {2, 5 }, 7, 9) is:

(a) 3

(b) 5

(c) 4

(d) 16

**Q2**. If A, B, C are three sets, then (A ∩ B) - C is:

(a) (A - C) ∪ (A - B)

(b) (A - C) ∩ (B- C)

(c) (A - B) ∪ (A - C)

(d) (A - B) ∩ (A - C)

**Q3**. If A-B=C and B-A=D, then C-D=

(a) C ∩ D

(b) D

(c) C

(d) C ∪ D

**Q4**. (A - B) ∪ (B - A) =

(a) (A ∪ B) - (A ∩ B)

(b) (A ∩ B) - (A ∪ B)

(c) A ∩ B

(d) A ∪B

**Directions for Questions (Q5&Q6):** In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers.

**Q5.** Find the number of people who read at least one of the news papers?

(a) 32

(b) 23

(c) 43

(d) 52

**Q6**. Find the Number of people who read exactly one news paper?

(a) 10

(b) 8

(c) 12

(d) 30

**Directions for questions (Q7-Q9):** Select the correct alternative from the given choices.

**Q7.** If A∪ B=C ∪ B and A ∩ B=C ∩ B, then

(a) A ⊆B

(b) A ∩ C=𝝓

(c) A = B

(d) A = C

**Q8.** If n(A) = 5, the number of subsets of set A is

(a) 25

(b) 52

(c) 55

(d) None of these

**Q9.** At a luncheon table where 12 men are sealed, one half of the men belong to club A, one third belong to club B, and one-fourth belong to both clubs. How many men belong to neither?

(a) 1

(b) 3

(c) 5

(d) 6

**Q10.** A market research group conducted a survey of 1000 consumers and reported that 720 consumers like product A and 450 consumers like product B, what is the least number that must have liked both products?

(a) 280

(b) 220

(c) 170

(d) 550

**Directions for questions (Q11-Q13)**: In a group of 15, 7 can speak Spanish, 8 can speak French and 3 can speak neither.

**Q11.** How many can speak both Spanish and French?

(a) 0

(b) 2

(c) 3

(d) 4

**Q12.** How many can speak only Spanish?

(a) 7

(b) 4

(c) 3

(d) 8

**Q13.** How many can speak at least one of the two languages from Spanish and French?

(a) 3

(b) 12

(c) 8

(d) 7

**Directions for questions (Q14-Q16):** In a survey of 200 students of higher secondary school, it was found that 120 study Mathematics, 90 study Physics and 70 study Chemistry.40 study Mathematics and Physics, 30 study Physics and Chemistry, 50 study Chemistry and Mathematics and 20 study none of the subjects.

**Q14.** Find the number of students who studied the 3 subjects?

(a) 10

(b) 20

(c) 30

(d) 40

**Q15.** How many studied exactly one subject?

(a) 100

(b) 110

(c) 90

(d) 120

**Q16.** How many studied at least two subjects?

(a) 80

(b) 90

(c) 70

(d) 40

**Q17.** In a class of 35 students, 24 like to play cricket and 16 like to play football. Also, each student likes to play at least one of the two games. How many students like to play both cricket and football?

(a) 15

(b) 9

(c) 7

(d) 5

**Q18.** In a survey of 400 students in a school, 100 were listed as taking apple Juice, 150 as taking orange juice and 75 were listed as taking both apple as well as orange juice. Find how many students were taking neither apple juice nor orange juice?

(a) 150

(b) 225

(c) 175

(d) 325

**Q19.** There are 200 individuals with a skin disorder, 120 had been exposed to the chemical C1, 50 to chemical C2, and 30 to both the chemicals C1 and C2. Find the number of individuals exposed to chemical C1 or chemical C2?

(a) 140

(b) 150

(c) 170

(d) 80

**Q20.** In a school there are 20 teachers who teach Mathematics or Physics. Of these, 12 teach Mathematics and 4 teach both Physics and Mathematics. How many teach physics?

(a) 4

(b) 8

(c) 12

(d) 10

**Q21.** A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of the three sports ?

(a) 10

(b) 20

(c) 43

(d) 9

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| 2.6 Additional Exercise from various competitive examinations |

**Directions for Questions (Q1 & Q2):** New Age Consultants have three consultants Gyani, Medha and Buddhi. The sum of the number of projects handled by Gyani and Buddhi individually is equal to the number of projects in which Medha is involved. All three consultants are involved together in 6 projects. Gyani works with Medha in 14 projects. Buddhi has 2 projects with Medha but without Gyani, and 3 projects with Gyani but without Medha. The total number of projects for New Age Consultants is one less than twice the number of projects in which more than one consultant is involved.

**Q1**. What is the number of projects in which Gyani alone is involved?

(a) 0

(b) 1

(c) 4

(d) Cannot be determined

**Q2.** What is the number of projects in which Medha alone is involved?  
(a) 0

(b) 1

(c) 4

(d) Cannot be determined

**Q3.** In a survey it was found that 40% like tea, 50% like coffee and 60% like milk. Every person likes at least one of the three items tea/coffee/milk. What are the maximum and minimum possible values of those who like all three?

(a) 20, 0

(b) 20, 10

(c) 25, 10

(d) 25, 0

**Q4.** Each item is followed by two statements, A and B. Answer each questions using the following instructions.

**Choose 1 if the question can be answered by one of the statements alone but not by the other.**

**Choose 2 if the question can be answered by using either statement alone.**

**Choose 3 if the question can be answered by using both the statements together, but cannot be answered by us either statement alone.**

**Choose 4 if the question cannot be answered by either of the statements.**

People in a club either speak French or Russian or both. Find the number of people in a club who speak only French?

**A.** There are 300 people in the club and the number of people who speak both French and Russian is 196.

**B.** The number of people who speak only Russian is 58.

(a) 1

(b) 2

(c) 3

(d) 4

**Q5.** If both a and b belong to set {1, 2, 3, 4}, then the number of equations of the form having real roots is

(a) 10

(b) 7

(c) 6

(d) 12

**Directions for Questions (Q6&Q7):** Let S be the set of all pairs (i, j) where 1 ≤ i < j ≤ n and n ≥ 4. Any two distinct members of S are called “friends” if they have one constituent of the pairs in common and “enemies” otherwise. For example, if n = 4, then S = {(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)}. Here, (1, 2) and (1, 3) are friends, (1, 2) and (2, 3) are also friends, but (1, 4) and (2, 3) are enemies.

**Q6.** For general n, how many enemies will each member of S have?

(a)

(b) 2n-7

(c)

(d)

(e) n-3

**Q7**. For general n, consider any two members of S that are friends. How many other members of S will be common friends of both these members?

(a) 2n-6

(b)

(c) n-2

(d)

(e)

**Q8.** Consider set S = {2, 3, 4, . . . , 2n+1},

where n is the positive integer larger than 2007. Define X as the average of the odd integers in S and Y is the average of the even integers in S. What is the value of X-Y?

(a) 1

(b)

(c)

(d) 2008

(e) 0

**Q9.** A guidance counselor is planning schedules for 30 students. Sixteen students say they want to take French, 16 want to take Spanish, and 11 want to take Latin. Five say they want to take both French and Latin, and of these, 3 wanted to take Spanish as well. Five want only Latin, and 8 want only Spanish. How many students want French only?

(a) 14

(b) 7

(c) 1

(d) 5

**Directions for Questions (Q10 &Q11):** All the members of a group of 30 teenagers belong to at least one club. There are 3 clubs, chess, drama and art. 6 of the teenagers belong to only the art club. 5 of the teenagers belong to all 3 clubs. 2 of the teenagers belong to the chess and art clubs but not to the drama club. 15 of the teenagers belong to the art club. 2 of the teenagers belong only to the chess club.3 of the teenagers belong only to the drama club

**Q10.** How many of the group does chess and drama but not art?

(a) 10

(b) 15

(c) 20

(d) 25

**Q11**. How many of the group belongs to the chess club?

(a) 15

(b) 19

(c) 25

(d) 29

**Directions for Questions (Q12&Q13):** Answer the following questions based on the information given below.

In a college library, four different business newspapers - Economic Times, Business Standard, Business Line and Financial Express - are available. All students visit the library regularly but 20% of them do not read any business newspaper. The four newspapers given in the above order are read by 230; 180; 180 &220 students respectively. The number of students reading exactly 2 newspapers for any two newspapers is 20. There are 30 students who read all the four news papers but there is nobody who reads exactly three out of four newspapers.

**Q12**. How many students do not read any newspaper at all?

(a) 50

(b) 40

(c) 90

(d) 150

**Q13**. What percentage of the people reading Business Standard also read

at least one other newspaper?

(a) 30%

(b) 40%

(c) 50%

(d) 90%

**Q14.** If all the students in the college including those who do not read any newspaper read at least one newspaper, (Out of the four newspapers above) which he is not reading at present, then what is the least number of students reading all the four newspapers?

(a) 10

(b) 30

(c) 50

(d) 70

**Q15.** Suppose I discovered that my cat had a taste for the adorable little geckoes that live in the bushes and vines in my yard, back when I lived in Arizona. In one month, suppose he deposited the following on my carpet: six gray geckoes, twelve geckoes that had dropped their tails in an effort to escape capture, and fifteen geckoes that he'd chewed on a little. Only one of the geckoes was gray, chewed on, and tailless; two were gray and tailless but not chewed on; two were gray and chewed on but not tailless. If there were a total of 24 geckoes left on my carpet that month, and all of the geckoes were at least one of "gray", "tailless", and "chewed on", how many were tailless and chewed on but not gray?

(a) 10

(b) 6

(c) 3

(d) 5

**Q16**. In a class of 120 students numbered 1 to 120, all even numbered students opt for Physics, whose numbers are divisible by 5 opt for Chemistry and those whose numbers are divisible by 7 opt for Math. How many opt for none of the three subjects?

(a) 19

(b) 41

(c) 21

(d) 26

**Q17**. Of the 200 candidates who were interviewed for a position at a call center, 100 had a two-wheeler, 70 had a credit card and 140 had a mobile phone. 40 of them had both, a two-wheeler and a credit card, 30 had both, a credit card and a mobile phone and 60 had both, a two wheeler and mobile phone and 10 had all three. How many candidates had none of the three?

(a) 0

(b) 20

(c) 10

(d) 18

**Q18.** The schedule of G first year students was inspected. It was found that M were taking a Mathematics course, L were taking a Language course and B were taking both a Mathematics course and a Language course. Which of the following expression gives the percentage of the students whose schedule was inspected who were taking neither a mathematics course nor a language course?

**Directions for Questions (Q19&Q20):** Answer the following questions based on the information given below.

There are five sets of digits - Set A, Set B, Set C, Set D and Set E as shown in given diagram. Set A contains one digit, Set B contains two digits, Set C contains three digits, Set D contains two digits and Set E contains one digit. Rearrange the digits, across the sets such that the number formed out of digits of Set C is multiple of the numbers formed from digits in the sets on either side. For example; in the given diagram, Set C is a multiple of digits in Set A and Set B but not of Set D and Set E.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| SET A | SET B | SET C | SET D | SET E |
| 7 | 28 | 196 | 34 | 5 |

**Q19**. What is the minimum number of rearrangements required to arrive at the solution? A rearrangement is defined as an exchange of positions between digits across two sets. For example: when 1 from set C is exchanged with 5 of set E, it is counted as one rearrangement.

(a) 2

(b) 5

(c) 8

(d) 3

(e) 7

**Q20**. Which of the following pair of digits would occupy set A and E?

(a) 2 and 4

(b) 2 and 6

(c) 3 and 6

(d) 3 and 9

(e) 4 and 8

**Q21.** A survey on a sample of 25 new cars being sold at a local auto dealer was conducted to see which of the three popular options “air conditioning, radio and power windows” were already installed. The survey found15 had air conditioning, 2 had air conditioning and power windows but no radios, 12 had radio 6 had air conditioning and radio but no power windows 11 had power windows,4 had radio and power windows and 3 had all three options.

What is the number of cars that had none of the options?

(a) 4

(b) 3

(c) 1

(d) 2

**Summary:**

A set is a well defined collection of objects.

A set which does not contain any element is called empty set.

A set which consists of a definite number of elements is called finite set, Otherwise, the set is called infinite set.

Two sets A and B are said to be equal if they have exactly the same elements.

A set A is said to be subset of a set B, if every element of A is also an element of B. Intervals are subsets of R.

A power set of a set A is collection of all subsets of A. It is denoted by P(A).

The union of two sets A and B is the set of all those elements which are either in A or in B.

The intersection of two sets A and B is the set of all elements which are common.

The difference of two sets A and B in this order is the set of elements which belong to A but not to B.

The complement of a subset A of universal set U is the set of all elements of U which are not the elements of A.

For any two sets A and B, (A ∪ B)′ = A′ ∩ B′ and ( A ∩ B )′ = A′ ∪ B′

If A and B are finite sets such that

A ∩ B = 𝝓, then

n (A ∪ B) = n (A) + n (B).

If A ∩ B ≠ 𝝓, then

n (A ∪ B) = n (A) + n (B) – n (A ∩ B)

**Chapter 3**

|  |
| --- |
| Equations - Simple & Quadratic and  Inequations |

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| 3.1 Learning objective: |

To learn about the basics of Equations and its Applications.

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| 3.2 Introduction: |

**Equation**

An equation is a math sentence that says that two things are equal. An equation always has an equal (=) sign. The thing or things that are on the left side of the equal sign are equal to the things on the right side of the equal sign. Here are a few equations:

X = Y + 5

A = B - 4

Equations are relations used to find the values of unknown quantities. These unknown quantities are known as variables. Variables are denoted by letters like x, y, a, b.

For example, if we have a statement saying that a person's age after 10 years becomes 50 years and if we form an equation for this, x+10=50. x represents a person’s age.

**Basic Operations on Variables:**

**Addition/Subtraction:**

3p + 5p = 8p Say 3 pens + 5 pens = 8 pens.

**Multiplication:**

Find the product of (4x + y) and (2x + 3y).

(4x + y) (2x + 3y) = 4x (2x + 3y) + y (2x + 3y) = 8x2 + 12xy + 2xy + 3y2 = 8x2 + 14xy + 3y2.

**Division:**

Divide (10a + 15) by 5.

**Factorization:**

An algebraic expression can be expressed as a product of two or more factors.

Factorize 9x2 +3xy=3x (3x+y)

Alternately: 5x+ 2 = 12 ⟹ 5x= 12 -2

5x = 10; x =10/5 = 2 .The value of x is 2.

**Identical Equations:**

Example: x+ y= 3, 2x+ 2y= 6 we cannot find the unique values of x and y because, if we try to evaluate one of the variables, both the variables will get cancelled.

If x = 2, y = 1, x + y = 3 and 2x + 2y = 6. If x=3, y=0, x+ y= 3 and 2x + 2y= 6.

If x = 11/2, y =11/2, Thus the equations x + y = 3 and 2x + 2y = 6, will have infinite set of solutions.

**Inconsistent Equations:**

**An equation that has no solution is called an inconsistent equation.**

It is easy to find equations that are false no matter what number we use to replace the variable. Consider the equation

x=x+1.

If we replace x by 3, we get 3=3+1, which is false. If we replace x by 4, we get 4=4+1, which is also false. Clearly, there is no number that will satisfy x=x+1.

**Conditional Equations:**

The statement 2x +4 =10 is true only on condition that we choose x=3. The equation x2=4 is satisfied only if we choose x=2 or x=-2. These equations

are called conditional equations.

**A conditional equation is an equation that is satisfied by at least one real number but is not an identity**

**Example 1:** Find the values of x and y if

4x + 3y = 10 and 7x - 3y = 1.

**Explanatory Answer:**

By adding both the equations we get,

11x =11. x = 1 in the first equation substitute x=1

4 x 1+ 3y=10

4 + 3y= 10

4 + 3y-4 = 10-4

3y=6.

3y=6

y = 2

x= 1, y= 2

**Example 2:** Find the solution for the equations 3x + 4y = 2 and 5x + 2y = 8.

**Explanatory Answer:**

3x + 4y = 2-----(i)

5x + 2y = 8------(ii)

(ii)\*2=>10x+4y=16---(iii)

(i)-(iii)⟹x=2.

Substitute x=2 in 1, we can get y=-1

**Example 3:** A plumber is paid $ for p hours and $y for every additional hour he works. On a certain day, the plumber earns $ . For how many hours did he work on that day?

**Explanatory Answer:**

Total amount earned =

=$

In order to earn $ the plumber worked for p hours. In order to earn $3y he worked for 3 hours. Total number of hours for which the plumber has worked (p + 3) hours.

**Quadratic Equations:**

An algebraic equation in which the highest power of the unknown is two and the powers of the unknowns are whole numbers, is known as a quadratic equation.

**Example 4:** ;

;

The standard form of a quadratic equation is , where a ≠ 0.

**Methods to Solve Quadratic Equation:**

Below are the four most commonly used methods to solve quadratic equations.

* The Quadratic Formula (Quadratic formula in depth)
* Factoring (Factoring Method in depth)
* Completing the Square
* Synthetic division

**The Quadratic Formula:**

The solution of a quadratic equation is the value of x when you set the equation equal to zero. i.e. 0 = ax² + bx + c

Given a quadratic equation: ax ² + bx + c

The quadratic formula below will solve the equation for zero

**Factoring:**

**Examples 5:** y = x² + 4x + 4

**Explanatory Answer:**

y = x² + 4x + 4

**Step1:**

Create factor chart

|  |  |
| --- | --- |
| Factors of C(as pairs) | Sum of factors |
| 1 , 4 | 1+4=5 |
| 2 , 2 | 2+2=4 |
| -1,-4 | -1 + -4-5 |
| -2, -2 | -2 + -2= -4 |

**Step2:** Determine which of factor pair of ‘c’ has a sum of ‘b’

**Step3:** Insert that pair in to y=(x+2)(x+2)

Binomial factors

**Step4:** solve each binomial x+2=0

For zero ⟹x=-2

**Completing the Square:**

**Step1:** Divide all terms by **a** (the coefficient of **x2**).

**Step2:** Move the number term (**c/a**) to the right side of the equation.

**Step3**: Complete the square on the left side of the equation and balance this by adding the same value to the right side of the equation. We now have something that looks like (x + p)2 = q, which can be solved rather easily.

**Step4**: Take the square root on both sides of the equation.

**Step5:** Subtract the number that remains on the left side of the equation to find **x**.

**Example 6:** Solve x2 + 4x + 1 = 0

**Explanatory Answer:**

**Step1:** Can be skipped in this example since the coefficient of x2 is 1

**Step2**: Move the number term to the right side of the equation:

x2 + 4x = -1

**Step3:** Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation.

(b/2)2 = (4/2)2 = 22 = 4

x2 + 4x + 4 = -1 + 4

(x + 2)2 = 3

**Step4**: Take the square root on both sides of the equation:

x + 2 = ± = ±1.73 (to 2 decimals)

**Step5:**  Subtract 2 from both sides:

x = ±1.73 – 2 = -3.73 or -0.27

It gives us a way to find the last term of a perfect square trinomial.

A perfect square trinomial is a polynomial that you get by squaring a.(binomials are things like 'x + 3' or 'x − 5')

Examples:

* x2+2x+1 This is a perfect square trinomial because it equals (x+1)2
* x2+4x+4 This is a perfect square trinomial because it equals (x+2)2
* x2+6x+9 This is a perfect square trinomial because it equals (x+3)2

**These are NOT perfect square trinomials:**

x2+3x+2 This is not a perfect square trinomial because it equals (x+1)(x+2)

(x+3)2=x2+6x+9=x2+?(3)+3?=x2+2(3)+32

### Factoring By Grouping:

If you have a quadratic equation in the form ax2+bx+c

**Step1:** Determine the product of **ac** (the coefficients in a quadratic equation)

**Step2:** Determine what factors of **ac** sum to **b**.

**Step3:** "ungroup" the middle term to become the sum of the factors found in step 2

**Step4:**  Group the pairs.

**Synthetic division:**

**Synthetic division** is a shortcut for polynomial division when the divisor is of the form x – a. Only numeric coefficients of the dividend are used when dividing with synthetic division.

##### **Example 7:** Divide (2 x – 11 + 3 x 3) by

##### ( x – 3).

**Step1:** Write the polynomial being divided in descending order. Then write only its coefficients and constant, using 0 for any missing terms.

|  |  |  |  |
| --- | --- | --- | --- |
| 3 | 0 | 2 | -11 |

**Step2:** Write the constant, a, of the divisor, x – a, to the left. In this problem, a = 3 because you use the additive inverse of the constant. (Remember, the additive inverse of –3 is 3)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 3 | 3 | 0 | 2 | -11 |

**Step3:** Bring down the first coefficient as shown.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 3 | 3 | 0 | 2 | -11 |
|  |  |  |  |  |
|  | 3 |  |  |  |

**Step4:** Multiply the first coefficient by the divisor, 3. Then write this product under the second coefficient.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 3 | 3 | 0 | 2 | -11 |
| (3 3=9 ) | | |  |  | 9 |  |  |
|  |  |  |  | 3 |  |  |  |

**Step5:** Add the second coefficient with the product and write the sum as shown.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 3 | 3 | 0 | 2 | -11 |
|  | | |  |  | 9 |  |  |
|  |  |  |  | 3 | 9 |  |  |

**Step 6:** Continue this process of multiplying and adding until there is a sum for the last column.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 3 | 3 | 0 | 2 | -11 |
|  | | |  |  | 9 | 27 | 87 |
|  |  |  |  | 3 | 9 | 29 | 76 |

The numbers along the bottom row are the coefficients of the quotient with the powers of x in descending order. The last coefficient is the remainder. The first power is one less than the highest power of the polynomial that was being divided.

The division answer is

**Minimum or Maximum value of a quadratic expression:**

The quadratic expression in the form of

Will have a minimum or maximum value when x = -b/a .

**Case1:** If a > 0, will have a minimum value.

**Case2:** If a < 0, will have a maximum value.

The minimum or maximum value is

|  |
| --- |
| 3.3 Quiz |

**Q1.** The perimeter of a rectangular plot of land is 32 m. If the length is increased by 2m and breadth is decreased by 1 m, the area of the plot remains the same. Find the length and breadth of the plot?

**Q2**. Three years ago Atul's age was four times Parul's age. After 5 years from now, Atul's age will be two times Parul's age. Find their present ages?

**Q3.** Two places A and B are 100 km apart. One car starts from A and another from B at the same time. If they travel in the same direction, they meet after 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars? Assume that the speed of car at A is more than the speed of car at B.

**Q4.** The sum of the digits of a two-digit number is 11. If the digits are reversed, the new number is 27 less than the original number. Find the original number?

**Q5**. Present ages of Reena and Meena are in the ration 4:5. After 8 years, the ratio of their ages will be 5:6. Find their present ages?

|  |
| --- |
| 3.4 Worked out examples |

**Q1**. Solve x² + 2x + 1=0

**Explanatory Answer:**

0= x² + 2x + 1

=>0=(x+1) (x+1)

=> x=-1

**Q2.** Find the missing value to complete the square x2+16x+?

**Explanatory Answer:**

**Step1:** Divide the middle term by 2.

**Step2:** Then square the result

82=64

From Step2 we get, x2+16x+64

So Answer is 64.

**Q3.** Solve x² + 6x + 9=0

**Explanatory Answer:**

0= x² + 6x + 9

=>0=x2+3x+3x+9

=>0=(x+3) (x+3)

=>x=-3 or x=-3

**Q4.** Solve 3x2+8x+4=0

**Explanatory Answer:**

The form of Given equation is

ax2+bx+c=0

Product of (a)(c) = (3)(4) = 12

What factors of 12 sum to 8?

2 & 6

Think of 8x as 2x + 6x

3x² + 2x + 6x + 4

Group the 2 pairs: (3x² + 2x) + (6x + 4)

Remove the common factors: x(3x+2)+2(3x+2)

Rewrite as grouped factors: (x+2)(3x+2)x=-2, -

**Q5**. Solve x2 + x + 1= 0

**Explanatory Answer:**

Here, b2 – 4ac= 12 – 4 × 1 × 1

= 1 – 4 = – 3

Therefore, the solutions are given by

**Q6**. Solve x2 + x + = 0

**Explanatory Answer:**

Here, the discriminant of the equation is

12 − 4× × = 1 – 20 = – 19

Therefore, the solutions are

**Q7**. Divide (5 x 4 + 6 x 3 – 9 x 2 – 7 x + 6) by ( x + 2) using the synthetic method.

**Explanatory Answer:**

To put the divisor, x + 2, into the form

x – a, use the constant's negative. That

means using x – (+2), so a = –2.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | -2 | 5 | 6 | -9 | -7 | 6 |
|  | | |  | -10 | 8 | 2 | 10 |
|  |  |  | 5 | -4 | -1 | -5 | 16 |

The answer is

### Q8. Solve 5x2 – 4x – 2 = 0

**Explanatory Answer:**

**Step1:** Divide all terms by 5

x2 – 0.8x – 0.4 = 0

**Step 2** : Move the number term to the right side of the equation:

x2 – 0.8x = 0.4

**Step3**: Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation:

(b/2)2 = (0.8/2)2 = 0.42 = 0.16

x2 – 0.8x + 0.16 = 0.4 + 0.16

(x – 0.4)2 = 0.56

**Step4**: Take the square root on both sides of the equation:

x – 0.4 = ± = ±0.748 (to 3 decimals)

**Step5**: Subtract (-0.4) from both sides (in other words, add 0.4):

x = ±0.748 + 0.4 = -0.348 or 1.148

**Q9.** The present age of Jacob’s father is three times that of Jacob. After 5 years, the difference of their ages will be 30 years. Find their present ages?

**Explanatory Answer:**

Let the present age of Jacob be x years.

Therefore, the present age of his father is 3x years.

After 5 years, the age of Jacob = (x + 5) years.

After 5 years, the age of his father = (3x + 5) years.

The difference of their ages = (3x + 5) – (x + 5) years, which is given to be

30 years, therefore 3x + 5 – (x + 5) = 30

3x + 5 – x – 5 = 30

or 3x – x = 30

or 2x = 30

or x = 15

Therefore, the present age of Jacob is 15 years and the present age of his father = 3x

= 3 × 15 = 45 years.

**Q10.**  Asha is five years older than Robert. Five years ago, Asha was twice as old as Robert was then. Find their present age?

**Explanatory Answer:**

Let present age of Asha be x years

and present age of Robert be y years

Therefore, x = y + 5

or x – y = 5 ...(1)

5 years ago, Asha was x – 5 years and Robert was (y – 5) years old.

Therefore, x – 5 = 2(y – 5)

or x – 2y = – 5 ...(2)

Solving (1) and (2),

we get y = 10 and x = 15

**Q11.** The perimeter of a rectangular garden is 20 m. If the length is 4 m more than the breadth, find the length and breadth of the garden?

**Explanatory Answer:**

Let the length of the garden be x m. Therefore, breadth of garden = (x – 4) m.

Since, perimeter is 20 m, so

2 [x + (x –4)] = 20

On solving, we get x = 7

length = 7 m and

breadth = 7 – 4 = 3 m.

**Q12**. The product of digits of a two digit number is 12. When 9 is added to the number, the digits interchange their places. Determine the number?

**Explanatory Answer:**

Let the digit at tens place be x

and digit at units place be y

Therefore, number = 10 x + y

When digits are interchanged, the number becomes 10y + x

Therefore 10x + y + 9 = 10y + x

or 10x – x + y – 10y = – 9

or 9x – 9y = –9

or x = y – 1 ...(1)

Also, product of digits is 12

Hence, xy = 12 ...(2)

Substituting value of x from (1) into (2),

We get (y – 1)y = 12

or y2 – y – 12 = 0

or (y – 4) (y + 3) = 0

y = 4 or y = – 3

Since, digit cannot be negative, y = 4

Hence x = y – 1 = 4 – 1 = 3

Therefore, the number is 34.

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| 3.5 Class Work Problems |

**Q1.** Solve x² − 6x + 9 =0

**Q2.** Solve x² + 2x – 3=0

**Q3.** Solve x2 + 5x + 6=0

**Q4.** Solve x2-x-12=0

**Q5.** Solve x2+15x+30=-6

**Q6.** Solve 11a2-32a+17=20

**Q7.** Solve 5x2-11x-3=2x+3

**Q8.** Solve 3x3+21x2+36x=0

**Q9.** Solve 2a3-18a2+36a=0

**Q10.** Solve x4-13x2+36=0

**Q11.**  Solve x4+3x2-4=0

**Q12.** Find the dimension of the rectangle below:

x+7

520 in2

2x

**Q13.** Find the missing value to complete the square

**1.** x2 + 20x +?

**2.** x2+18x+?

**3.** x2+7x+?

**Q14.** Solve 2x² + 5x + 3

**Q15.** Solve x2 - x + 3=0

**Q16.** Solve

**Q17.** Solve

**Q18.** Divide

by (x+4)

(a) 2 x 4 + 3 x 3 + x 2 – 2 x

(b) 2 x 3 – 3 x 2 + x + 2

(c) 2 x 3 + 3 x 2 + x – 2

**Q19.** Find the value of x in

(a)

(b)

(c)

(d)

**Q20.** If =79, then find

(a) 7

(b) 8

(c) 11

(d) 9

**Q21**. A ball is shot into the air from the edge of a building, 50 feet above the ground. Its initial velocity is 20 feet/sec.

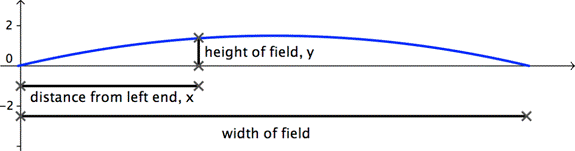
The Equation h=-16t2+20 t+50 can be used to model the height of the ball after t seconds. How long does it take for the ball to hit the ground?

**Q22**. We are going to fence in a rectangular field and we know that for some reason we want the field to have an enclosed area of 75 ft2.  We also know that we want the width of the field to be 3 feet longer than the length of the field.  What are the dimensions of the field?

**Q23**. Two cars start out at the same point.  One car starts out driving north at 25 mph.  Two hours later the second car starts driving east at 20 mph.  How long after the first car starts traveling does it take for the two cars to be 300 miles apart?

**Q24**. An office has two envelope stuffing machines.  Working together they can stuff a batch of envelopes in 2 hours.  Working separately it will take the second machine 1 hour longer than the first machine to stuff a batch of envelopes.  How long would it take each machine to stuff a batch of envelopes by themselves?

**Q25.** A stadium field of synthetic turf appears to be flat, its surface is actually shaped like a parabola. This is so that rainwater runs off to the sides. If we take a cross section of the turf, the surface can be modeled by, where *x* is the distance from the left end of the field and *y* is the height of the field. What is the width of the field?



(a) 80 ft

(b) 1.5 ft

(c) 234 ft

(d) 160 ft

**Q26**. A ball is launched upward at 48 ft/s from a platform that is 100 ft. high. Find the maximum height the ball reaches and how long it will take to get there?

**Q27**. A farmer has 1000 feet of fencing and a very big field. She can enclose a rectangular area with dimensions x ft and (500 – x) ft. What is the largest rectangular area she can create?

(a) 62,500 ft2

(b) 250,000 ft2

(c) 1,000 ft2

(d) 500 ft2

**Q28.**

|  |  |
| --- | --- |
| **Selling Price $ (s)** | **Quantity Sold in 1 year (q)** |
| 10 | 1000 |
| 15 | 900 |
| 20 | 800 |
| 25 | 700 |

the cost to produce each item is $10

Using the data given above, determine the selling price(s), which produce the maximum yearly profit ?

**Q29**. Bob made a quilt that is 4 ft x 5 ft.  He has 10 sq. ft. of fabric to create a border around the quilt. How wide should he make the border to use all the fabric? (The border must be the same width on all four sides.)

**Q30.** The following price vs. Quantity information.

|  |  |
| --- | --- |
| Selling Price s | Quantity Sold q |
| 100 | 7000 |
| 200 | 6000 |
| 500 | 3000 |
| 600 | 2000 |
| 800 | 0 |

Write an equation that will represent yearly profit P for a selling price s. The production cost per item is $30.

(a) P = -10s + 8000

(b) P = sq – 30q

(c)

(d) P

**Q31**. Arial has an 8 x 10 photo that she would like to frame. She would like the total area of the photo and frame to equal 143 square inches. If the width of the frame is the same on all sides, what is the width of the frame?

**Q32**. A ball is thrown at a speed of 24 feet per second from a height of 22 feet. What is the maximum height the ball will reach?

After how many seconds will it hit the ground?

**Q33.** An online book company offers

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| --- | --- |
| **Delivery Option** | **Total Cost Equation** |
| 3 - 5 business days | C = 3 + 0.99x |
| Within 2 business days | C = 9.99 + 1.99x |

different shipping rates for different delivery options. The equations shown above can be used to calculate the total shipping cost, C, for a delivery of x

number of books.

Alisha ordered 5 books from the company. How much money does she save, in dollars, if she chooses the first delivery option over the second?

**Q34**. Luca's drive to college includes a 260-mile stretch on an interstate highway and at least one hour of driving time off the highway. The maximum speed limit on the highway is 65 miles per hour. The equation below shows the relationship between distance, speed, and time.

Distance = Speed x Time

Assuming Luca drives at a constant speed that is not over the speed limit on the highway, what is Luca's minimum driving time, in hours?

**Q35**. John is eleven times as old as his son Marcus. In twelve years, John will be three times as old as Marcus. How old is Marcus now?

**Q36.** Find two numbers such that their sum is 21 and their product is 104?

**Q37.** The sum of a number and its reciprocal is . Find the number?

**Q38.** If the length and width of a 4- by 2-inch rectangle are each increased by the same amount, the area of the new rectangle will be twice that of the original. What are the dimensions of the new rectangle (to decimal places)?

**Q39**. Find the base b and height h of a

triangle with an area of 2 square feet if its

base is 3 feet longer than its height and the formula for area is A bh

**Q40**. At what interest rate will $1,000 increase to $1,440 in 2 years?

**Q41**. Two planes travel at right angles to each other after leaving the same airport at the same time. One hour later they are 260 miles apart. If one travels 140 miles per hour faster than the other, what is the rate of each?

**Q42**. A speedboat takes 1 hour longer to go 24 miles up a river than to return. If the boat cruises at 10 miles per hour in still water, what is the rate of the current?

**Q43**. One pipe can fill a tank in 5 hours less than another. Together they can fill the tank in 5 hours. How long would it take each alone to fill the tank?

**Q44.** Two gears rotate so that one completes 1 more revolution per minute than the other. If it takes the smaller gear 1 second less than the larger gear to complete revolution, how many revolutions does each gear make in one

minute?

**Q45.** A shot-put throw can be modeled using the equation, where x is distance traveled (in feet) and y is the height (also in feet).  How long was the throw?

**Q46**. Find all numbers with the property

that when the number is added to itself the sum is the same as when the number is multiplied by itself?

**Chapter 4**

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| Percentage, Ratio and Proportion |

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| 4.1 Learning objective: |

To learn about the basics of Percentage, Ratio and Proportion and its Applications.

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| 4.2 Introduction: |

**Ratio:**

A ratio is simply a fraction.   The following notations all express the ratio of x to y

⟹  x:y , x÷y , or   
In the ratio x : y, we call a as the first term or **antecedent** and b, the second term or **consequent***.*

Writing two numbers as a ratio provides a convenient way to compare their sizes.   For example, since, we know that 3 is less than 4.

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**Rule:** The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

**Example:** 3: 5 = 6 : 10 = 12 : 20.

**Proportion:**

The equality of two ratios is called proportion.

If a : b = c : d, we write a : b **::** c : d and we say that a, b, c, d are in proportion.

Here a and d are called **extremes**, while b and c are called **mean terms.**

**Product of means = Product of extremes**

Thus, a: b: c: d http://www.indiabix.com/_files/images/aptitude/1-sym-bim.gif(b x c) = (a x d).

**Fourth Proportional:**

If a: b = c : d, then d is called the fourth proportional to a, b, c.

**Third Proportional:**

a: b = c : d, then c is called the third proportion to a and b.

**Mean Proportional:**

Mean proportional between a and b is ab.

**Comparison of Ratios:**

**We say that (a:b)>(c:d) ⟺**

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**Compounded Ratio:**

The compounded ratio of the ratios: (a : b), (c : d), (e : f) is (ace : bdf).

**Duplicate Ratios:**

Duplicate ratio of (a : b) is (a2 : b2).

Sub-duplicate ratio of (a : b) is (a : b).

Triplicate ratio of (a : b) is (a3 : b3).

Sub-triplicate ratio of (a : b) is (a1/3 : b1/3).

Componendo and Dividendo

If then,

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**Variations:**

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We say that x is directly proportional to y, if x = ky for some constant k and we write, x 𝛂 y.

We say that x is inversely proportional to y, if xy = k for some constant k and

we write, x 𝛂

**Percentage:**

By a certain percent, we mean that many hundredths.

Thus, x percent means x hundredths, written as x%.

To express x% as a fraction: We have,

x% =

Thus, 40% =

To express as a percent: We have,

)%

**Percentage Increase/Decrease:**

If the price of a commodity increases by X%, then the reduction in consumption so as not to increase the expenditure is:

100 %

If the price of a commodity decreases by R%, then the increase in consumption so as not to decrease the expenditure is:

100 %

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**Results on Population:**

Let the population of a town be P now and suppose it increases at the rate of R% per year, then:

Population after n years =

Population n years ago =

**Results on Depreciation:**

Let the present value of a machine be P. Suppose it depreciates at the rate of R% per year. Then:

Value of the machine after n years =

Value of the machine n years ago =

If A is x% more than B, then B is less than A by [ 100] %

If A is x% less than B, then B is more than A by [ 100] %

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| 4.3 Quiz |

**Q1**. Find the fourth proportional to the numbers 6, 12 and 15?

**Q2.** If a: b = 2:3, b: c = 5:7 c:d = 4:3, find a:b:c:d?

**Q3.** Two numbers are respectively 30% and 40% less than a third number. What percent is the second of the first?

**Q4.** If the annual increase in the population of the town be 5% and the present population be 25000, what was it two years ago?

**Q5.** If the price of the petrol is increased by 10%, find how much percentage must reduce in consumption of petrol, so as not to increase the expenditure?

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| 4.4 Worked out examples |

**Q1.** Three friends divide Rs.624 in the ratio . Then, the share of the third friend is:

(a) Rs.288

(b) Rs.192

(c) Rs.148

(d) Rs.144

The correct choice is (d) and the correct answer is Rs.144.

**Explanatory Answer:**

= 6: 4 : 3

The share of the third friend =

**Q2.** If then (5x-4y):(3x+2y) is

(a) 21 : 2

(b) 21 : 13

(c) 2 : 21

(d) 13 : 21

The correct choice is c. and the correct answer is 2:21

**Explanatory Answer:**

**Q3.** A bag contains 50 paise, 25 paise and 10 paise coins in the ratio 5: 9: 4 amounting to Rs.206. The number of 25 paise coins is:

(a) 200

(b) 360

(c ) 160

(d) 260

The correct choice is (b) and the correct answer is 360

**Explanatory Answer:**

The number of 25 paise coins must be a multiple of 9. The only multiple of 9 in the given answers is 360

**Q4**. The incomes of A, B, C are in the ratio of 7: 9: 12 and their spending are in the ratio of 8: 9: 15. If A saves of his income, then the savings of A, B, C are in the ratio of?

(a) 56: 99: 69

(b) 99: 56: 69

(c) 69: 56: 99

(d) 99: 69: 56

The correct choice is (a) and the correct answer is 56: 99: 69

**Explanatory Answer:**

Let the incomes be 7x, 9x, 12x

Spending be 8y, 9y, 15y

Then, savings will be 7x -8y, 9x – 9y, 12x- 15y

The middle part has to be a multiple of 9,

Correct option is (a).

**Q5.** 94 is divided into two parts in such a way that fifth part of the first and eighth part of the second are in the ratio 3 : 4. The first part is:

(a) 27

(b) 30

(c) 36

(d) 48

The correct choice is (b) and the correct answer is 30

**Explanatory Answer:**

(F/5): (S/8) = 3: 4

4F / 5 = 3S /8

32 F =15S

F: S = 15:32

30:64 to make the sum 94

**Q6.** A 24 litres mixture contains water and milk in the ratio 3: 5. How much water should be added to this mixture to reverse this ratio?

(a) 10

(b) 12

(c) 15

(d) 16

The correct choice is (d) and the correct answer is 16

**Explanatory Answer:**

W : M

3 : 5

9 : 15

to make the sum 24

9 + x : 15 = 5 : 3

27 + 3x = 75;

x = 16

**Q7**. In an election contested by two parties, Party A secured 12% of the total votes more than Party B. If party B got 132,000 votes, by how many votes did it lose the election?

(a) 300, 000

(b) 168, 000

(c) 36, 000

(d) 24, 000

The correct choice is (c) and the correct answer is 36000

**Explanatory Answer:**

Let the percentage of the total votes secured by Party A be x%

Then the percentage of total votes secured by Party B=(x–12) %

As there are only two parties contesting in the election, the sum total of the votes secured by the two parties should total up to 100%

i.e., x+x–12=100

2x–12=100

or 2x=112 or x%=56%.

If Party A got 56% of the votes, then Party B got (56 – 12) = 44% of the total votes.

44% of the total votes = 132,000

i.e., (44100) ×T=132,000

⇒ T= (132,000×10044)=300,000 votes.

The margin by which Party B lost the election = 12% of the total votes

= 12% of 300,000 = 36,000

**Q8.** If the price of petrol increases by 25% and Raj intends to spend only an additional 15% on petrol, by how much % will he reduce the quantity of petrol purchased?

(a) 10%

(b) 12%

(c) 8%

(d) 6.67%

The correct choice is (c) and the correct answer is 8%

**Explanatory Answer:**

Let the price of 1 litre of petrol be Rs.x and let Raj initially buys 'y' litres of petrol.

Therefore, he would have spent Rs. xy on petrol.

When the price of petrol increases by 25%, the new price per litre of petrol is 1.25 x.

Raj intends to increase the amount he spends on petrol by 15%.

i.e., he is willing to spend xy + 15% of xy = 1.15xy

Let the new quantity of petrol that he can get be 'q'.

Then, 1.25x q = 1.15xy

⇒ q=

As the new quantity that he can buy is 0.92y, he gets 0.08y lesser than what he used to get earlier or a reduction of 8%.

**Q9**. A batsman scored 110 runs which included 3 boundaries and 8 sixes. What percent of his total score did he make by running between the wickets?

(a) 45%

(b) %

(c) 54%

(d) 55%

The correct choice is (b) and the correct answer is %

**Explanatory Answer:**

Number of runs made by running =110−(3×4+8×6)

= 110 - (60)

= 50.

Percentage= (×100)%

=%

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| 4.5 Class Work Problems |

**Q1**. If A: B = 2 : 3 and B : C = 4 : 5 then C : A is:

(a) 15 : 8

(b) 12 : 10

(c) 8 : 5

(d) 8 : 15

**Q2**. The diameters of the fore wheel and the rear wheel of a tractor are in the ratio 1: 4. When the fore wheel makes 100 revolutions per minute, how many revolutions per minute would the rear wheel make?

(a) 18

(b) 23

(c) 25

(d) 17

**Q3.** In factory men, women and children were employed in the ratio 8:5:1to finish a job and their individual wages were in the ratio 5:2:3. Total daily wages of all amount to Rs.318. Find the total daily wages paid to each category.

(a) Rs.240, 60, 18

(b) Rs.210, 70, 38

(c) Rs.190, 95, 33

(d) None of these

**Q4.** Rs.53 is divided among A, B, C such that A’s share is Rs.7 more than B’s share and B’s share is Rs.8 more than C’s share. The ratio of their shares is:

(a) 16: 9: 18

(b) 25: 18: 10

(c) 18: 25: 10

(d) 15: 8: 30

**Q5.** From each of two given numbers, half the smaller number is subtracted Of the resulting numbers the larger one is three times as large as the smaller. What is the ratio of the two numbers?

(a) 2:1

(b) 3:1

(c) 3:2

(d) None of these

**Q6**. R divides two sums of money among his four sons G, M, A and S. The first sum is divided in the ratio 4: 3: 2: 1 and second in the ratio 5: 6: 7: 8. If the second sum is twice the first, the largest total is received by

(a) G

(b) M

(c) A

(d) S

**Q7.** A stockiest wants to make some profit by selling sugar. He contemplates about various methods. Which of the following would maximize his profit?

I. Sell sugar at 10% profit?

II. Use 900g of weight instead of 1 Kg.

III. Mix 10% impurities in sugar and selling sugar at cost price.

IV. Increase the price by 5% and reduce weights by 5%.

(a) I or III

(b) II

(c) II, III and IV

(d) Profits are same

**Q8.** A man buys spirit at Rs. 60 per litre, adds water to it and then sells it at Rs.75 per litre. What is the ratio of spirit to water if his profit in the deal is 37.5%?

(a) 9:1

(b) 10:1

(c) 11:1

(d) None of these

**Q9.** There are two containers: the first contains 500ml of alcohol, while the second contains 500ml of water. Three cups of alcohol from the first container is taken out and is mixed well in the second container. Then, three cups of this mixture is taken out and is mixed in the first container. Let A denote the proportion of water in the first container and B denote the proportion of alcohol in the second container. Then,

(a) A > B

(b) A < B

(c) A = B

(d) Cannot be determined

**Q10**. A dealer buys dry fruits at Rs. 100, Rs. 80 and Rs. 60 per kilogram. He mixes them in the ratio 3:4:5 by weight, and sells at a profit of 50%. At what price per kilogram does he sell the dry fruit?

(a) Rs. 80

(b) Rs. 100

(c) Rs. 95

(d) None of these

**Q11**. Instead of a metre scale, a cloth merchant uses a 120 cm scale while buying, but uses an 80 cm scale while selling the same, cloth. If he offers a

discount of 20% on cash payment, what is his overall profit percentage?

(a) 20%

(b) 25%

(c) 40%

(d) 15%

**Q12.** The cost of diamond varies directly as the square of its weight. Once, this diamond broke into four pieces with weights in the ratio 1:2:3:4. When the pieces were sold, the merchant got Rs. 70,000 less. Find the original price of the diamond.

(a) Rs.1.4 lakh

(b) Rs.2 lakh

(c) Rs. 1 lakh

(d) 2.1 lakh

**Q13**. You can collect rubies and emeralds as many as you can. Each ruby is of Rs. 5 crore. Each ruby weighs 0.3 kg. And each emerald weighs 0.4 kg. Your bag can carry at the most 12 kg. What you should collect to get the maximum wealth?

(a) 20 rubies and 15 emeralds

(b) 40 rubies

(c) 28 rubies and 9 emeralds

(d) None of these

**Q14.** I have one rupee coins, fifty paise coins and twenty five paise coins are in the ratio 2.5 : 3 : 4. If the total amount with me is Rs. 210, find the number of one rupee coins.

(a) 90

(b) 85

(c) 100

(d) 105

**Q15.** Instead of walking along two adjacent sides of a rectangular field, a boy took a short cut along the diagonal and saved a distance equal to half the longer side. Then, the ratio of the shorter side to

the longer side is :

(a) ½

(b) 2/3

(c) ¼

(d) ¾

**Q16.** Mayank, Mirza, Little and Jaspal bought a motorbike for $60,000. Mayank paid one half of the sum of the amounts paid by the other boys; Mirza paid one third of the sum of the amounts paid by the other boys; and Little paid one fourth of the sum of the amounts paid by the other boys. How much did Jaspal has to pay?

(a) 15

(b) 13

(c) 17

(d) None of these

**Q17.** Fresh grapes contain 90% water by weight by while dried grapes contain 20% water by weight. What is the weight of dry grapes available from 20 kg of fresh grapes?

(a) 2 kg

(b) 2.4 Kg

(c) 2.5 kg

(d) None of these

**Q18.** Using only 2, 5, 10, 25 and 50 paise coins, what will be the minimum number of coins required to pay exactly 78 paise, 69 paise and Re 1.01 to three different persons?

(a) 19

(b) 20

(c) 17

(d) 18

**Q19.** A milkman mixes 20 litres of water with 80 litres of milk. After selling one-fourth of this mixture, he adds water to replenish the quantity that he has sold. What is the current proportion of water to milk?

(a) 2 : 3

(b) 1 : 2

(c) 2 : 3

(d) 3 : 4

**Q20.** A report consists of 20 sheets each of 55 line and each line consists of 65 characters. This percentage reduction in number of sheets is closet to

(a) 20

(b) 5

(c) 30

(d) 35

(e) 35

**Q21.** The rate of increase of the price of sugar observed to be two percent more than the inflation are expressed in percentage. The price of sugar, on January 1, 1994 is Rs. 20 per kg. The inflation rates of the years 1994 and 1995 are expected to be 8% each. The expected price of sugar price on January 1, 1996 would be

(a) Rs. 23.60

(b) Rs. 24.00

(c) Rs. 24.20

(d) Rs. 24.60

**Q22.** The number of votes not cast for the Praja Party increased by 25% in the National General Election over those not

cast for it in the previous Assembly Polls, and the Praja Party lost by a majority twice as that by which it had won the Assembly Polls. If a total 2,560,000 people voted each time, how many voted for the

Praja Party in the previous Assembly Polls?

(a) 1,10,000

(b) 1,50,000

(c) 1,40,000

(d) 1,20,000

**Q23.** 2/5 of the voters promise to vote for P and the rest promised to vote for Q. Of these, on the last day 15% of the voters went back of their promise to vote for P and 25% of voters went back of their promise to vote for Q, and P lost by 2 votes. Then, the total number of voters is:

(a) 100

(b) 110

(c) 90

(d) 95

**Q24.** Forty per cent of employees of a certain company are men and 75% of the men earn more than Rs. 25,000 per year. If 45% of the company’s employees earn more than Rs. 25,000 per year, what fraction of the women employees by the company earn Rs. 25,000 per year.

(a)

(b)

(c)

(d)

**Q25**. A College has raised 75% of the amount it needs for a new building by receiving and average donation of Rs. 600 from the people already solicited. The people already solicited represent 60% of the people, the college will ask for donations. If the college is to raise exactly the amount needed for the new building, what should be the average donation from the remaining people to be solicited?

(a) 300

(b) 250

(c) 400

(d) 500

**Q26.** A student took five papers in an examination, where the full marks were the same for each paper. His marks in these papers were in the proportion of 6 : 7 : 8 : 9 : 10. In all papers together, the candidate obtained 60% of the total marks. Then, the number of papers in which he got more than 50% marks is:

(a) 1

(b) 3

(c) 4

(d) 5

**Q27.** A owner of an art shop conducts his business in the following manner. Every once in a while he raises his prices by X% then a while later he reduces all the new prices by X%. After one such up-down cycle, the price of a painting decreased by Rs. 441. After a second up-down cycle, the painting was sold for Rs. 1944.81. What was the original price of the painting (in Rs.)?

(a) 2756.25

(b) 2256.25

(c) 2500

(d) 2000

**Q28.** At the end of year 1998, S bought nine dozen goat. Henceforth, every year he added p% of the goats at the beginning of the year and sold q% of the goats at the end of the year where p> 0 and q > 0. If S had nine dozen goats at the end of the year 2002, after making the sales for that year, which of the following is true?

(a) p = q

(b) p < q

(c) p > q

(d) p = q /2

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| 4.6 Additional Exercise from various competitive examinations |

**Directions for Questions (Q1-Q3):** Answers the questions based on the following information.

Alphonsa on his death bed, keeps half his property for this wife and decides the rest equally among his three sons : Ben, Carl and Dave, Some years later, Ben dies leaving half his property to his widow and half to this brothers Carl and Dave together, sharing equally. When Carl makes his will, he keeps half his property for his widow and the rest he bequeaths to his younger brother Dave. When Carl dies some years later, he keeps half his property for his widow and the remaining for his mother. The mother now has Rs. 1,575,000.

**Q1.** What was the worth of the total property?

(a) Rs. 30 Lakh

(b) Rs. 8 Lakh

(c) Rs. 18 Lakh

(d) Rs. 24 Lakh

**Q2**. What was Carl’s original share?

(a) 4 Lakhs

(b) 12 Lakhs

(c) 6 Lakhs

(d) 5 Lakhs

**Q3.** What was the ratio of the property owned by the window of the three sons, in the end?

(a) 7:9:13

(b) 8:10:15

(c) 5:7:9

(d) 9:12:13

**Q4.** Two oranges, three bananas and four apples cost Rs. 15. Three oranges, two bananas and one apple cost Rs. 10. I bought 3 oranges, 3 bananas and 3 apples. How much did I pay?

(a) Rs.10

(b) Rs. 8

(c) Rs. 15

(d) Rs. 15

(e) Cannot be determined

**Q5.** Two liquids A and B are in the ratio 5:1 in container 1 and 1:3 in container 2. In what ratio should the contents of the two containers be mixed so as to obtain a mixture of A and B in the ratio 1:1?

(a) 2:3

(b) 4:3

(c) 3:2

(d) 3:4

**Q6.** A person who has a certain amount with him goes to market. He can buy 50 oranges or 40 mangoes. He retains 10% of the amount for taxi fares and buys 20 mangoes and of the balance, he purchases oranges. Find Number of oranges he can purchase is:

(a) 36

(b) 40

(c) 15

(d) 20

**Q7.** The price of a Maruti car rises by 30% while the sales of the car comes down by 20%. What is the percentage change in the total revenue?

(a) -4%

(b) -2%

(c) +4%

(d) +2%

**Q8.** I bought 5 pens, 7 pencils and 4 erasers. Rajan bought 6 pens, 8 erasers and 14 pencils for an amount which was half more what I had paid. What per cent of the total amount paid by me was paid for the pens?

(a) 37.5%

(b) 62.5%

(c) 50%

(d) None of these

**Q9.** A man earns *x*% on the first Rs. 2,000 and y% on the rest of his income. If he earns Rs. 700 from Rs. 4,000 and Rs. 900 from Rs. 5,000 of income, find *x*%.

(a) 20%

(b) 15%

(c) 25%

(d) None of these

**Q10.** One obtains splits into eight bacteria of the next generation. But due to environment, only 50% of one generation can produce the next generation. If the seventh generation number is 4096 million, what is the number in first generation?

(a) 1 million

(b) 2 million

(c) 4 million

(d) 8 million

**Q11.** In a survey of political preferences, 78% of those were in favor of at least one of the proposals; I, II and III, 50% of those asked favored proposal I, 30% favored proposal II and 20% favored proposal II. If 5% of those asked favored proposals, what percentage of those asked favored more than one of the three proposals?

(a) 10

(b) 12

(c) 17

(d) 22

**Q12.** A survey on a sample of 25 new cars being sold at a local auto dealer was conducted to see which of the three popular options-air conditioning, ratio and power windows-were already installed. The survey found: 15 had air conditioning, 2 had air conditioning and radio but no power windows, 11 had power windows, 4 had radio and power windows, 3 had all three options. What is the number of cars that had none of the options?

(a) 4

(b) 3

(c) 1

(d) 2

**Directions for Questions (Q13& Q14):** Answer the questions on the basis of the information given below?

In an examination, there are 100 questions divided into three groups A, B & C carriers 3 marks. It is known that the questions in group A carries 1 mark, each question in group B carries 2 marks and each questions in group C carries 3 marks. It is known that the questions in group A together carry at least 60% of the total marks.

**Q13.** If group B contains 23 questions, then how many questions are there in group C?

(a) 4

(b) 2

(c) 1

(d) Cannot be determined

**Q14.** If group C contains 8 questions and group B carries at least 20% of the total marks, which of the following best describes the number of questions in group B?

(a) 11 or 12

(b) 12 or 13

(c) 13 or 14

(d) 14 or 15

**Q15.** The length, breath and height of a room are in the ratio 3:2:1. If the breadth and height of a room are in the ratio 3;2:1. If the breadth and height are halved while the length is doubled, then the total area of four walls of the rooms will

(a) remain the same

(b) decrease by 13.64

(c) decrease by 15%

(d) decrease by 30%

**Chapter 5**

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| Mixtures |

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| 5.1 Learning objective: |

To learn about the basic of Mixtures and its Applications.

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| 5.2 Introduction: |

**Alligation** :

A process or rule for the solution of problems concerning the compounding or mixing of ingredients differing in price or quality.

It is the rule that enables us to find

(a) The ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price.

(b) The average price of the mixture when prices of two or more ingredients and the proportion in which they are mixed is given.

**Mean Price:**

The cost price of a unit quantity of the mixture is called the mean price.

**Basic allegation formula:**

If two ingredients are mixed then

The above formula can be represented in diagram below

Ingredient A

(Price x)

Ingredient B

(Price y)

Mean

(Price M)

(M-y)

(x-M)

:

**Replacement of part of solution formula:**

Suppose a container contains x litres of a solution from which y quantity of solution is taken out and replaced with one of the ingredients. This process is repeated n times then, solution left in the vessel after the nth operation/Original quantity of solution in the vessel *=*

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| 5.3 Quiz |

**Q1.** In what ratio must a person mix three kinds of tea costing Rs.60/kg, Rs.75/kg and Rs.100/kg so that the resultant mixture when sold at Rs.96/kg yields a profit of 20%?

(a) 1 : 2 : 4

(b) 3 : 7 : 6

(c) 1 : 4 : 2

(d) None of these

**Q2.** How many litres of a 12 litre mixture containing milk and water in the ratio of

2: 3 be replaced with pure milk so that the resultant mixture contains milk and water in equal proportion?

(a) 4 litres

(b) 2 litres

(c) 1 litre

(d) 1.5 litres

**Q3.** From a cask of milk containing 30 litres, 6 litres are drawn out and the cask is filled up with water. If the same process is repeated a second, then a third time, what will be the number of litres of milk left in the cask?

(a) 0.512 liters

(b) 12 liters

(c) 14.38 liters

(d) 15.36 liters

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| 5.4 Worked out examples |

**Q1**. In what proportion must wheat at Price 4.10 per kg must be mixed with wheat at Price 4.60 per kg, so that the mixture be worth Rs 4.30 a kg?

**Explanatory Answer:**

Here, CP of unit quantity of Dearer = 460 per kg ,

CP of unit quantity of Cheaper = 410 per kg

So, the required ratio is 3:2

**Q2.** How many kg of rice at Rs. 60 per kg, must be be mixed with 30 kg of rice at Rs 25 per kg, so that he may on selling the mixture at Rs 50 per kg gain 25 % on the outlay?

**Explanatory Answer:**

First we have to find cost price of mixture, as seller is gaining 25 % profit on mixture so its cost price will be :

50 = C.P of the mixture × = C.P of the mixture = 40

Now use the formula of alligation to find out quantity of dearer rice,

Quantity of dearer = = 22.5 kgs

**Q3.** A jar contains a mixture of two liquids A and B in the ratio 4 : 1. When 10 litres of the liquid B is poured into the jar, the ratio becomes 2: 3. How many litres of liquid A were contained in the jar?

**Explanatory Answer:**

B in 1st

1/5

B in 2nd

1

3/5

1-(3/5)

(3/5) – (1/5)

:

The average composition of B in the first mixture is 1/5.

The average composition of B in the second mixture = 1

The average composition of B in the resultant mixture = 3/5

Hence applying the rule of Alligation we have

= =1

So, initial quantity of mixture in the jar = 10 litres.

And, quantity of A in the jar = = 8 litres.

**Q4.** Milk and water are mixed in a vessel A in the ratio 5 : 3 and in vessel B in ratio 9 : 7.In what ratio should quantities be taken from the two vessels so as to form a mixture in which milk and water will be in the proportion of 7 : 5?

**Explanatory Answer:**

In vessel A, milk = = of the weight of mixture

In vessel B, milk = = of the weight of mixture

Now, we have to form a mixture in which milk be of the weight of the mixture

Now according to rule of alligation :

5/8

9/16

7/12

(7/12)-(9/16)

( (5/8)–(7/12)

:

- =

- =

Required ratio = =

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| 5.5 Class Work Problems |

**Q1.** In what ratio must rice at Rs 9.30 per Kg be mixed with rice at Rs 10.80 per Kg so that the mixture be worth Rs 10 per Kg?

(a) 6:5

(b) 8:7

(c) 3:7

(d) 6:1

**Q2.** How much water must be added to 60 litres of milk at 1.5 litres for Rs. 20 so as to have a mixture worth Rs.10 a litre?

(a) 10 litres

(b) 12 litres

(c) 15 litres

(d) 18 litres

**Q3.** The milk and water in two vessels A and B are in the ratio 4:3 and 2:3 respectively. In what ratio the liquids in both the vessels be mixed to obtain a new mixture in vessel C consisting half milk and half water?

(a) 8 : 3

(b) 7 : 5

(c) 4 : 3

(d) 2 : 3

**Q4.** How many kilograms of sugar costing Rs. 9 per kg must be mixed with 27kg of sugar costing Rs.7 per kg so that there may be gain of 10% by selling the mixture at Rs.9.24 per kg?

(a) 60 kg

(b) 63 kg

(c) 50 kg

(d) 77 kg

**Q5.** A can contains a mixture of two liquids A and B in the ratio 7:5 when 9 litres of mixture are drawn off and the can is filled with B, the ratio of A and B becomes 7:9. How many litres of liquid A was contained by the can initially?

(a) 28 litres

(b) 21 litres

(c) 45 litres

(d) 36 litres

**Q6.** 8 litres are drawn from a cask filled with wine and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of the total solution is 16:81. How much wine did the cask hold originally?

(a) 24 litres

(b) 45 litres

(c) 49 litres

(d) 44 litres

**Q7.** A vessel is filled with liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?

(a) 711

(b) 67

(c) 15

(d) 27

**Q8.** A milk vendor has 2 cans of milk .The first contains 25% water and the rest milk. The second contains 50% water. How much milk should he mix from each of the container so as to get 12 litres of milk such that the ratio of water to milk is 3:5?

(a) 6 litres

(b) 1 litres

(c) 8 litres

(d) 7 litres

**Q9.** Tea worth Rs. 126 per kg and Rs. 135 per kg are mixed with a third variety in the ratio 1:1:2. If the mixture is worth Rs 153 per kg , the price of the third variety per kg will be?

(a)Rs. 147.50

(b)Rs. 785.50

(c)Rs. 175.50

(d)Rs. 258.50

**Q10.** A merchant mixes three varieties of rice costing Rs.20/kg, Rs.24/kg and Rs.30/kg and sells the mixture at a profit of 20% at Rs.30 / kg. How many kgs of the second variety will be in the mixture if 2 kgs of the third variety is there in the mixture?

(a) 1 kg

(b) 5 kgs

(c) 3 kgs

(d) 6 kgs

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| 5.6 Homework Problems |

**Q1.** A container contains 40 litres of milk. From this container 4 litres of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by the container?

(a) 26 litres

(b) 29.16 litres

(c) 28 litres

(d) 28.2 litres

**Q2.** Two vessels A and B contain spirit and water in the ratio 5 : 2 and 7 : 6 respectively. Find the ratio in which these mixture be mixed to obtain a new mixture in vessel C containing spirit and water in the ration 8: 5?

(a) 3: 4

(b) 4 : 3

(c) 9 : 7

(d) 7 : 9

**Q3.** A jar full of whiskey contains 40% alcohol. A part of this whisky is replaced by another containing 19% alcohols and now the percentage of alcohol was found to be 26%. The quantity of whisky replaced is

(a) 43

(b) 34

(c) 32

(d) 23

**Q4.** A 20 litre mixture of milk and water contains milk and water in the ratio 3: 2. 10 litres of the mixture is removed and replaced with pure milk and the operation is repeated once more. At the end of the two removals and replacement, what is the ratio of milk and water in the resultant mixture? (CAT)

(a) 17 : 3

(b) 9 : 1

(c) 3 : 17

(d) 5 : 3

**Q5.** How many kgs of Basmati rice costing Rs.42/kg should a shopkeeper mix with 25 kgs of ordinary rice costing Rs.24 per kg so that he makes a profit of 25% on selling the mixture at Rs.40/kg? (CAT)

(a) 20 kgs

(b) 12.5 kgs

(c) 16 kgs

(d) 200 kgs

**Q6**. How many litres of water should be added to a 30 litre mixture of milk and water containing milk and water in the ratio of 7 : 3 such that the resultant mixture has 40% water in it? (CAT)

(a) 7 litres

(b) 10 litres

(c) 5 litres

(d) None of these

**Q7.** A sample of x litres from a container having a 60 litre mixture of milk and water containing milk and water in the ratio of 2 : 3 is replaced with pure milk so that the container will have milk and water in equal proportions. What is the value of x? (CAT)

(a) 6 litres

(b) 10 litres

(c) 30 litres

(d) None of these

**Q8.** The ratio of milk to water in 66 kg of adulterated milk is 5: 1. Water is added to it to make this ratio 5: 3. Then what is the quantity of water added?

(a) 22 kg.

(b) 24.750 kg.

(c) 16.500 kg.

(d) 20 kg.

**Q9**. A merchant has 50 kg, of sugar, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. The quantity sold at 18% profit is:

(a) 20 kg.

(b) 30 kg.

(c) 15 kg.

(d) 35kg

**Q10.** A can contains a mixture of two liquids. A and B in proportion 7: 5. When 9 litres of mixture are drawn off and the can is filled with B, the proportion of A and B becomes 7: 9. How many litres of liquid A was contained by the can initially?

(a) 25

(b) 10

(c) 20

(d) 21

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| 5.7 Self Assessment |

**Q1.** Kantilal mixes 80 kg. of sugar worth of Rs. 6.75 per kg. with 120 kg. worth of Rs. 8 per kg. At what rate shall he sell the mixture to gain 20%?

(a) Rs. 7.50

(b) Rs. 9

(c) Rs. 8.20

(d) Rs. 8.85

**Q2.** A dishonest milkman professes to sell his milk at C.P. but he mixes it with water and thereby gains 25%. The percentage of water in the mixture is:

(a) 25%

(b) 20%

(c) 4%

(d) None of these

**Q3.** A can contains a mixture of two liquids. A and B in proportion 7: 5. When 9 litres of mixture are drawn off and the can is filled with B, the proportion of A and B becomes 7: 9. How many litres of liquid A was contained by the can initially?

(a) 25

(b) 10

(c) 20

(d) 21

**Q4.** In what ratio should water be mixed with milk to gain 16 % on selling the mixture at cost price?

(a) 6: 1

(b) 1 : 6

(c) 1: 4

(d) 4 : 1

**Q5.** 4 kg of a metal contains 1/5 copper and rest in Zinc. Another 5 kg of metal contains 1/6 copper and rest in Zinc. The ratio of Copper and Zinc in the mixture of these two metals

(a) 49 : 221

(b) 39:231

(c) 94:181

(d) None of these

**Q6.** A milk man sells the milk at the cost price but he mixes the water in it and thus he gains 9.09%.The quantity of water in the mixture of 1 liter is:

(a) 83.33 ml

(b) 90.90 ml

(c) 99.09 ml

(d) Cannot be determined

**Q7.** From a container, 6 liters milk was drawn out and was replaced by water. Again 6 liters of mixture was drawn out and was replaced by the water. Thus the quantity of milk and water in the container after these two operations is 9:16. The quantity of mixture is:

(a) 15

(b) 16

(c) 25

(d) 31

**Q8.** The ratio of petrol and kerosene in the container is 3:2 when 10 liters of the mixture is taken out and is replaced by the kerosene, the ratio become 2:3. Then total quantity of the mixture in the container is:

(a) 25

(b) 30

(c) 45

(d) Cannot be determined

**Q9**. In a mixture of milk and water, there is only 26% water. After replacing the mixture with 7 liters of pure milk, the percentage of milk in the mixture becomes 76%. The quantity of mixture is:

(a) 65 liters

(b) 91 liters

(c) 38 liters

(d) None of these

**Q10.** The diluted wine contains only 8 liters of wine and the rest is water. A new mixture, whose concentration is 30%, is to be formed by replacing wine. How many liters of mixture shall be replaced with pure wine if there was initially 32 liters of water in the mixture?

(a) 4

(b) 5

(c) 8

(d) None of these

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| 5.8 Additional questions |

**Q1.** If a certain quantity of 40 % HCL is mixed with 60% HCL in the ratio 4:1.This is mixed with 32% HCL in the ratio 2:1.From this 25 l is removed and replaced with 50 l water to get 20% HCL. What is the amount of 60% HCL (in litres) taken for the entire process?

**Q2.** Ashok a master adulterator cum grocer sells turmeric powder which contains 5% sawdust. What quantity of pure turmeric should be added to 2kg of adulterated turmeric so that the proportion of saw dust becomes 4%?

**Q3.** In a mixture of 35l, the ratio of petrol to alcohol is 4:1. If 7l of alcohol is added to the mixture the ratio of petrol to alcohol changes to a new ratio. If we want the ratio of petrol to alcohol to change back to the original value, how much petrol is to be added now?

**Q4.** A girl has 1 l of solution that contains orange juice and water in the ratio3:1. She adds 250ml of 3:2 solution of orange juice and water to it and then use 250ml of the combined mixture to make complete juice. How much juice is she left with?

**Q5.** Several litres of water were drawn from a 54 l vessel full of water and an equal amount of acid is added. Again the same volume of the mixture was drawn off and replaced by acid. As a result the vessel contained 24L of pure water. How much water was drawn off initially?

**Q6.** A 735 gm sample of a 16% (by weight) solution of antiseptic in water is kept for a week . Some of the water gets evaporated and the concentration of the solution becomes 20% (by weight). What amount of water gets evaporated?

**Q7.** A vessel contains 12l Vodka and another contains 4l whisky. 3l is removed from each of the vessels and transferred into the other. The same operation is done once more. What is the final ratio of vodka to whisky in two vessels?

**Q8.** How many kilograms of tea worth Rs 25 per kg must be blended with 30 kg of tea worth Rs 30 per kg so that by selling the blended variety at Rs 30 per kg there should be a gain of 10%?

**Q9.** One test-tube contains some acid and another test-tube contains an equal quantity of water. To prepare a solution, 20g of the acid is poured into the second test-tube. Then, two-third of the so-formed solution is poured from the second tube into the first. If the fluid in the first test-tube is four times that in the second, what quantity of water was taken initially?

**Q10.** There are two vessels of equal capacity, one full of milk, and second one-third full of water. The second vessel is, then filled up out of the first, the contents of the second are then poured back into the first till it is full and then again the contents of the first are poured back into the second till it is full. What is the proportion of milk in the second vessel, if capacity of the vessel is 20l ?

**Chapter 6**

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| Partnership |

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| 6.1 Learning objective: |

To learn about the basic of Partnership and its Applications.

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| 6.2 Introduction: |

**Partnership**:

When two or more than two persons run a business jointly, they are called partners and the deal is known as partnership.

**Ratio of Division of Gains:**

When investments of all the partners are for the same time period, the gain or loss is distributed among the partners in the ratio of their investments.

Suppose A and B invest Rs. x and Rs. y respectively for a year in a business, then at the end of the year:

(A’s share of profit):( B’s share of profit) = x : y.

When investments are for different time periods, then equivalent capitals are calculated for a unit of time by taking (capital × number of units of time). Now, gain or loss is divided in the ratio of these capitals.

Suppose A invests Rs. x for p months and B invests Rs. y for q months, then

(A’s profit share) :( B’s profit share)

=xp : yq.

**Working and Sleeping Partners**:

A partner who manages the business is known as working partner and the one who simply invests the money is a sleeping partner.

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| 6.3 Quiz |

**Q1.** Arun, Kamal and Vinay invested Rs. 8000, Rs. 4000 and Rs. 8000 respectively in a business. Arun left after six months. If after eight months, there was a gain of Rs. 4005, then what will be the share of Kamal? (MAT)

(a) Rs. 890

(b) Rs. 1335

(c) Rs. 1602

(d) Rs. 1780.

**Q2.** A, B and C started a business with investments in the ratio 1:2:4. After 6 months A invested half the amount more as before and B invested twice the amount as before while C withdrew 1/4th of their investments. Find the ratio of the profit at the end of the year. (FMS)

(a) 5:6:10

(b) 5:12:14

(c) 6:5:10

(d) 12:5:14

**Q3.** X, Y and Z divide Rs 120 among themselves. X’s share is 20 more than Y’s share. Z’s share is 20 more than X’s share. What is Y’s share? (IIFT)

(a) Rs 40

(b) Rs 30

(c) Rs 25

(d) Rs 20

**Q4.** Rahul started a business with a capital of Rs 8000. After 6 months Sanjay joined him with an investment of some capital. If at the end of the year each of them gets equal amount as profit, how much did Sanjay invest in the business?

(MAT)

(a) 18,000

(b) 17, 500

(c) 17,000

(d) 16, 500

**Q5**. X and Y start a business. X invests 3000 for 4 months and Y invests Rs 2000 for 6 months. How much should X be paid out of a total profit of Rs 500? (FMS)

(a) 200

(b) 300

(c) 250

(d) 350

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| 6.4 Worked out examples |

**Q1.** A, B and C start a business each investing Rs. 20,000. After 5 months A withdrew Rs. 5000, B withdrew Rs. 4000 and C invests Rs. 6000 more. At the end of the year, a total profit of Rs. 69,900 was recorded. Find the share of each?

**Explanatory Answer:**

Ratio of the capitals of A, B and C

= (20000 × 5 + 15000 ×7): (20000 × 5 + 16000 × 7): (20000 × 5 + 26000 × 7)

205000: 212000: 282000 = 205 : 212 : 282.

∴ A’s share = Rs. (69900 × 205/699)

= Rs. 20500;

B’s share = Rs. (69900 × 212/699)

=Rs. 21200;

C’s share = Rs. (69900 × 282/699)

= Rs. 28200.

**Q2.** Four milkmen rented a pasture. A grazed 24 cows for 3 months; B 10 cows for 5 months; C 35 cows for 4 months and D 21 cows for 3 months. If A’s share of rent is Rs. 720, find the total rent of the field.

**Explanatory Answer:**

Ratio of shares

A:B:C:D=(24×3): (10 × 5) : (35 × 4) : (21× 3)

= 72 : 50 : 140 : 63.

Let total rent be Rs. X.

Then, A’s share = Rs.

∴ = 720 ⇔ x = = 3250.

**Q3.** A, B and C started a business by investing Rs. 1,20,000, Rs. 1,35,000 and Rs 1,50,000 respectively. Find the share of each, out of an annual profit of Rs. 56,700.

**Explanatory Answer:**

Ratio of shares of A, B and C = Ratio of their investments

= 120000 : 135000 : 150000 = 8 : 9 : 10.

∴ A’s share = Rs. [= Rs. 16800.

B’s share = Rs. [] = Rs. 18900.

C’s share = Rs. [] = Rs. 21000.

**Q4.** A, B and C enter into a partnership and their shares are in ratio 1/2 : 1/3 : 1/4, after 2 months, A withdraws half of his capital and after 10 months, a profit of Rs 378 is divided among them. What is B's

share?

**Explanatory Answer:**

Ratio of investments =: :

now LCM of 2, 3, 4 is 12.

On multiplying the ratio by 12 we get 6 : 4 : 3 , also we assume their initial investment be 6x, 2x and 3x so, we can write:

A : B :C

=(6x × 2 + 3x × 10): (4x× 12): (3x×12)

=7:8:6

B’s share = × 378 = Rs. 144

**Q5.** A and B are partners in a business. At the end A’s capital was 1 / 4 of the total capital. A was in the business for 15 months. If B received 2 / 3 of the profit at the end, for how long B's money was used?

**Explanatory Answer:**

Let total profit is x

B’s share = , A’s share = ( -

Ratio of profit A : B = : = 1:2

Let total capital invested be Rs P and A's money was used for 15 months while B's money was used for b months then we can write the equation as-

So, B's money was used for 10 months

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| 6.5 Class Work Problems |

**Q1.** Albert started a business investing Rs. 45,000. After 3 months, Peter joined him with a capital of Rs. 60,000. After another 6 months, Ronald joined them with a capital of Rs. 90,000. At the end of the year, they made a profit of Rs. 16,500. Find Ronald’s share

(a) 3300

(b) 4400

(c) 4400

(d) 6600

**Q2.** A and B start a business jointly. A invests Rs.16000 for 8 months and B remains in the business for 4 months. Out of total profit B claims 2/7 of the profit. How much money was contributed by B ?

(a) 12,600

(b) 12,700

(c) 12,800

(d) 12,900

**Q3.** A and B started a business with initial investments in the ratio 14: 15 and their annual profits were in the ratio 7: 6. If A invested the money for 10 months, for how many months did B invest his money?

(a) 2

(b) 4

(c) 6

(d) 8

**Q4.** A starts business with Rs.3500 and after 5 months, B joins with A as his partner. After a year, the profit is divided in the ratio 2 : 3. What is B’s contribution in the capital?

(a) 6000

(b) 9000

(c) 9500

(d) 12000

**Q5.** A began a business with Rs. 85,000. He was joined afterwards by B with Rs. 42,500. For how much period does B join, if the profits at the end of the year are divided in the ratio of 3 : 1?

(a) 8 months

(b) 6 months

(c) 4 months

(d) 2 months

**Q6.** A, B and C started a shop by investing Rs. 27,000, Rs.72, 000 and Rs. 81,000 respectively. At the end of the year, the profits were distributed among them. If C’s share of profit be Rs. 36,000, then the total profit was:

(a) 30,000

(b) 60,000

(c) 80,000

(d) 120,000

**Q7.** A and B started a partnership business investing some amount in the ratio of 3:5. C joined them after six months with an amount equal to that of B. In what proportion should the profit at the end of one year be distributed among A, B and C?

(a) 5: 6: 10

(b) 6 : 10 : 5

(c) 6: 5: 10

(d) 10 : 6 : 5

**Q8**. A, B and C rent a pasture. A puts 10 oxen for 7 months, B puts 12 oxen for 5 months and C puts 15 oxen for 3 months for grazing. If the rent of the pasture is Rs. 175, how much must C pay as his share of rent?

(a) 45

(b) 50

(c) 55

(d) 65

**Q9.** A and B entered into partnership with capitals in the ratio 4: 5. After 3 months, A withdrew ¼ of his capital and B withdrew 1/5 of his capital. At the end of 10 months, the gain was Rs. 760. What is A's share in the profit?

(a) 310

(b) 330

(c) 370

(d) 350

**Q10.** A and B started a business jointly. A’ s investment was thrice the investment of B and the period of his investment was two times the period of investment of B. If B received Rs.4000 as profit, then their total profit is:

(a) 21,000

(b) 23,000

(c) 20,000

(d) 25,000

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| 6.6 Homework Problems |

**Q1.** Three partners, A, B, C invest Rs.36000, Rs 45000, and Rs.54000 respectively in a business Out of a total profit of Rs.37500, What is C’s share?

**Q2.** Kavitha and Sunitha are partners in a business. Kavitha invests Rs.35000 for 8 months and Sunitha invests Rs.42000 for 10 months. Out of a profit of Rs.31570, What is Kavitha’s share?

**Q3.** Jayant opened a shop investing Rs.30000, Madhu joined him 2 months later investing Rs.45000. They earned a profit of Rs.54000 after completion of one year. What will be Madhu’s share of profit?

**Q4**. Nirmal and Kapil started a business investing Rs.9000 and Rs.12000 respectively. After 6 months, Kapil withdrew half of his investment. If after a year, the total profit was Rs.4600, what was Kapil's share in it?

**Q5**. A,B and C enter into a partnership. A initially invests Rs.25 lakh and adds another Rs.10 lakhs after one year. B initially invests Rs.35 lakh and withdraws Rs.10lakh after 2 years and C invests Rs.30 lakh. In what ratio should the profits be divided at the end of 3 years?

**Q6.** Mohinder and Surinder entered into a partnership investing Rs.12000 and Rs.9000 respectively. After 3 months Sudhir joined them with an investment of Rs.15000. What is the share of Sudhir in a half yearly profit of Rs.9500?

**Q7.** A and B entered into a partnership investing Rs.16000 and Rs.12000 respectively. After 3 months A withdrew Rs.5000 while B invested Rs.5000 more. After 3 more months C joins the business with a capital of Rs.21000. Out of a total profit of Rs.26,400 after one year how much more is B’s share than that of C?

**Q8.** X and Y invested in a business. They earned some profit which they divided in the ratio of 2:3. If X invested Rs.40000, What is the amount invested by Y ?

**Q9.** Manoj received Rs.6000 as his share out of the total profit of Rs.9000 which he and Ramesh earned at the end of one year. If Manoj invested Rs.20,000 for 6 months, whereas Ramesh invested his amount for the whole year , what was the amount invested by Ramesh ?

**Q10.** Three partners A,B,C start a business. Twice A’s capital is equal to thrice B’s capital and B’s capital is four times C’s capital. Out of a total profit of Rs.16,500 . What is B’s share at the end of the year?

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| 6.7 Self Assessment problems |

**Q1**. A, B, C subscribe Rs.50000 for a business. A subscribes Rs.4000 more than B and B Rs.5000 more than C. Out of a total profit of Rs.35000 A receives (MAT)

(a) 15,000

(b) 14, 700

(c) 14, 000

(d) 13, 700

**Q2.** Rs.700 is divided among A, B, C so that A receives half as much as B and B half as much as C. Then C’s share is:

(a) 200

(b) 400

(c) 600

(d) 800

**Q3.** A,B,C, hire a meadow for Rs.1095, if A

puts in 10 cows for 20 days B 30 cows for 8 days and C 16 cows for 9 days, then the rent paid by C is:

(a) 270

(b) 300

(c) 325

(d) 350

**Q4.** Four milkmen rented a pasture. A grazed 24 cows for 3 months; B 10 cows for 5 months, C 35 cows for 4 months and D 21 cows for 3 months. If A’s share of rent is Rs.720, the total rent of the field.

(a) 3050

(b) 3150

(c) 3250

(d) 3350

**Q5.** A, B and C enter into partnership. A invests some money at the beginning , B invests double the amount after 6 months and C invests thrice the amount after 8 months. If the annual profit be Rs.27, 000, C’s share is:

(a) 6000

(b) 9000

(c) 7000

(d) 1000

**Q6.** In a partnership, A invests 1/6 of the capital for 1/6 of the time; B invests 1/3 of the capital for 1/3 of the time and C, the rest of the capital for the whole time. Out of a profit of Rs.4600, what is B’s share?

**Q7.** A, B, C started a business with their investments in the ratio 1:3:5 .After 4 months. A invested the same amount as before and B as well as C withdrew half of their investments. What is the ratio of their profits at the end of the year?

**Chapter 7**

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| Data Interpretation |

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| 7.1 Learning objective: |

To learn about the Applications of Data Interpretation

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| 7.2 Introduction: |

Data Interpretation has no particular syllabus. It tests your speed, decision making capability and analysing data. It consists of a good number of graphs, charts and tables from which you will have to analyse data. The key to cracking this area is to quickly identify the key pieces of data that you will require to work on the questions asked. It is not unknown for question-setters of the CAT to try and bewilder students with a large amount of data, most of it unnecessary. As a rule, the more the data presented, the easier the questions that follow, so don't lose heart if you see a table with 10 columns occupying one whole page. On the other hand, several seemingly innocuous questions may trip you up.

Get familiar with all types of DI questions: When you have been practicing DI questions for a while, you'll begin to notice that there are different types or formats of questions in this section. There will be bar charts, tables, pie charts etc. There will also be questions where a lot of text/information will be provided in 1 long paragraph. Also try reading business newspapers which have articles that are data-intensive & have graphs & pie-charts etc. After reading the article, try to mentally collate & analyse the data provided in them. Practice, practice, practice: With a very good amount of practice your speed and confidence will increase considerably.

**Pie Charts**

They derive their name from its shape, like that of a pie divided into various portions. They always represent data in the form of a percentage of the total, with the total percentage being 100. In such a chart, the length of the arc (and therefore the angle each sector subtends at the centre) is proportional to the quantity it represents. Such charts are often used in the corporate world and in newspapers. Since a circle comprises 360 degrees, each percent of a pie-chart is equal to 360 divided by 100, or 3.6 degrees. This fact will be important for the calculations you are expected to perform.

**Bar Graphs**

Bar graphs represent data in the form of columns or bars. Bar graphs can be horizontal or vertical. The length of the bar is proportional to the data value

represented by it.

**Line Graphs**

Line graph represents data in the form of straight lines that connect various data values. Both line graphs and bar graphs are used to convey same things and hence can be used inter-changeably. For example, a line graph can be generated by joining the tip of the bar graph.

**Caselets**

In caselets, the mathematical data is represented in the form of a paragraph. Hence extracting data and establishing relationships between different data values becomes difficult. However caselets are very popular with the CAT examiners.

**Combined Data Sets**

Data is represented in two or more different types of data sets. It could be combination of a table and a graph or two or more graphs. You may have to correlate the data in different data sets to solve these questions. Thus interpret ting data takes time. These type of sets are very common in GRE. However since CAT is going online, there is a good chance that these sets may figure in CAT as well. However our gut feel is that if such a set comes in CAT, then it would not be heavy on data and be an easy set to interpret because of the limited space on the computer screen.

**Tips for Solving DI Problems:**

**A.** Read the data very carefully, as the smallest detail may change the meaning of the question completely. Similarly, the instructions have to be understood carefully to prevent wasting time in calculating data that is not required, and also to find out exactly what is the answer that is sought.

**B.** Try to understand the data provided carefully, before jumping to answer the questions. The questions are designed to be deceptive, and proper understanding of the requirements is a must. If the Data provided is of the combined variety or if there are more than one data table/charts/graphs, try to understand the relation between the given tables. For Example, one table may talk about absolute sales figures, while the other table may talk of sales as a percentage of production. Hence, any question on excess production or Goods in stock, will require data from both tables.

**C.** Be very careful of the units used in the tables, and the units in which the answers (options) are provided. A mistake in the units may yield an entirely different answer. Also be careful of whether the answer is required in decimal or percentage. Such errors are common and easily avoidable.

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| 7.3 Worked out examples |

**Directions for Questions (Q1- Q3):**

The table below provides certain demographic details of 30 respondents who were part of a survey. The demography characteristics are: gender, number of children, and age of respondents. The first number is each cell is the number of respondents in that group. The minimum and maximum age of respondents in each group is given in brackets. For example, there are 5 female respondents with no children and among these five; the youngest is 34 years old, while the oldest is 49.

|  |  |  |  |
| --- | --- | --- | --- |
| **No of Children** | **Male** | **Female** | **Total** |
| 0 | 1 (38, 38) | 5 (34, 49) | 6 |
| 1 | 1 (32, 32) | 8 (35, 57) | 9 |
| 2 | 8 (21, 65) | 3 (37, 63) | 11 |
| 3 | 2 (32, 33) | 2 (27, 40) | 4 |
| Total | 12 | 18 | 30 |

**Q1.** The percentage of respondents aged less than 40 years is at least.

(a) 10%

(b) 16.67%

(c) 20.0%

(d) 30%

**Explanatory Answer:**

The condition “number of respondents aged less than 40 years is at least” can be interpreted as:

* If minimum and maximum age in a group <40, then all respondents in the group meet the condition
* If minimum age <40 and maximum >40, then at least 1 respondent meets the condition
* If both minimum and maximum ages >40, then no respondent meets the condition

|  |  |  |  |
| --- | --- | --- | --- |
| No. of  children | Male | Female | Total  = 9 |
| 0 | 1 | 1 |
| 1 | 1 | 1 |
| 2 | 1 | 1 |
| 3 | 2 | 1 |

Therefore, number of respondents meeting the above criteria is 9 as shown in the Table above. Hence 30% is the correct answer.

**Q2.** Given the information above, the percentage of respondents older than 35 can be at most.

(a) 30%

(b) 73.33%

(c) 76.67%

(d) 90%

**Explanatory Answer:**

|  |  |  |  |
| --- | --- | --- | --- |
| No. of  children | Male | Female | Total  = 23 |
| 0 | 1 | 4 |
| 1 | 0 | 7 |
| 2 | 7 | 3 |
| 3 | 0 | 1 |

The condition “maximum number of respondents older than 35” can be

interpreted as:

* If minimum and maximum age <= 35, then no respondent meets the condition
* If minimum <= 35 and maximum > 35, then one less than the number of respondents in the group meet the condition
* If minimum and maximum > 35 then, all respondents meet the condition

Therefore, correct answer is 76.67%.

**Q3.** The percentage of respondents that fall into the 35 to 40 years age group (both inclusive) is at least

(a) 6.67%

(b) 10%

(c) 13.33%

(d) 26.67%

**Explanatory Answer:**

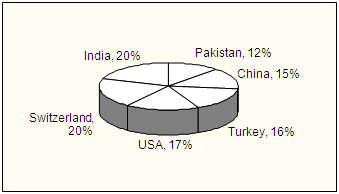
The condition “minimum number of respondents that fall into the 35 to 40 years age group (both inclusive)” can be interpreted as:

|  |  |  |  |
| --- | --- | --- | --- |
| No. of children | Male | Female | Total = 4 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 2 | 0 | 1 |
| 3 | 0 | 1 |

* If both minimum and maximum ages are < 35 or > 40 then no respondent meets the condition
* If both minimum and maximum ages are 35 to 40 then all respondents meet the condition
* If minimum < 35 and maximum between 35 to 40 then at least 1 respondent meets the condition
* If minimum is between 35 to 40 and maximum > 40 then at least 1 respondent meets the condition

**Directions for Questions (Q4& Q5):**

Answer the questions based on the pie charts given below:



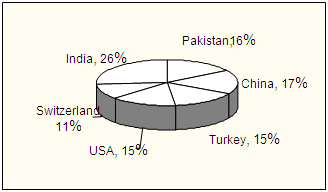


Chart-1 shows the distribution by value of top 6 suppliers of MFA Textiles in 1995. Chart-2 shows the distribution by quantity of top 6 suppliers of MFA textiles in 1995. The total value is 5760 million Euros (the European currency). The total

quantity is 1.055 million tones.   
  
**Q4.** The country, which has the highest average price, is:

(a) USA

(b) Switzerland

(c) Turkey

(d) India

**Explanatory Answer:**

Conventional approach to solve the problem is to compute the average price of all 4 countries and compare the values-

USA =

Switzerland =

Turkey =

India =

You would notice that all values contain a factor of.This factor can be ignored because you are required to identify the country having the highest average value, not the average value itself. Hence, you have to just compare , , and . Obviously, Switzerland gets the highest average price. Smart thinkers can easily solve the problem by observation without writing even a single number on paper.

**Q5.** The average price in Euro/ kg for Turkey is roughly

(a) 6.20

(b) 5.60

(c) 4.20

(d) 4.80

**Explanatory Answer:**

The question can also be answered by some quick mental calculations. The overall average price for all countries is about 5.5 (). For Turkey, share of value to share of quantity is , which is slightly more than 1. Average price of Turkey should be a multiplication of these two ratios. Hence, the answer will be roughly 5.60

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| 7.4 Class Work Problems |

**Direction for Questions (Q1- Q4):**

A health-drink company’s R & D department is trying to make various diet

formulations, which can be used for certain specific purposes.

It is considering a choice of 5 alternative ingredients (O, P, Q, R, and S), which can be used in different proportions in the formulations. The table below gives the composition of these ingredients. The cost per unit of each of these ingredients is O: 150, P: 50, Q: 200, R: 500, S: 100.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Composition | | | |
| Ingredients | Carbohydrate % | Protein % | Fat % | Minerals % |
| O | 50 | 30 | 10 | 10 |
| P | 80 | 20 | 0 | 0 |
| Q | 10 | 30 | 50 | 10 |
| R | 5 | 50 | 40 | 5 |
| S | 45 | 50 | 0 | 5 |

**Q1.** For a recuperating patient, the doctor recommended a diet containing 10% minerals and at least 30% protein. In how many different ways can we prepare this diet by mixing at least two ingredients? (a) One

(b) Two

(c) Three

(d) Four

**Q2.** Which among the following is the formulation having the lowest cost per unit for a diet having 10% fat and at least 30% protein? The diet has to be formed by mixing two ingredients. (a) P & Q

(b) P & S

(c) P & R

(d) Q & S

**Q3.** The company is planning to launch a balanced diet required for growth needs of adolescent children. This diet must contain at least 30% each of carbohydrate and protein, no more than 25% fat and at least 5% minerals. Which one of the following combinations of equally mixed ingredients is feasible? (a) O & P

(b) R & S

(c) O& S

(d) Q & R

**Q4.** In what proportion should P, Q and S be mixed to make a diet having at least 60% carbohydrate at the lowest per unit cost? (a) 2 : 1 : 3

(b) 4 : 1 : 1

(c) 2 : 1 : 4

(d) 3 : 1: 2

**Direction for Questions (Q5-Q8):**

128 players take part in a Grand Slam Tennis tournament. The tournament is scheduled to be held in seven rounds and in each round, in a match between two players, the winner advances to the next round and the loser is eliminated. There are no draws or byes in the tournament. The players who take part in the tournament are seeded from 1 to 128, with seed 1 being the top seed, seed 2 next and so on. The matches are scheduled in such a way that in any round, assuming there are no upsets, the highest seeded player plays against the lowest seeded player at that point, the next highest seeded player always plays against the next lowest seeded player and so on. An upset is said to happen when a lower seeded player beats a higher seeded player. The schedule of matches in the next round remains unchanged in case of an upset in a round, with the only difference that the player who caused the upset advances to the next round and takes the designated place of the player he upset.

**Q5.** In case of no upsets in the tournament, in which round would the player seeded 10 face a player seeded higher than him?

(a) 2nd round

(b) 3rd round

(c) 4th round

(d) 5th round

**Q6**. How many players in the tournament won exactly one match?

(a) 15

(b) 24

(c) 30

(d) None of these

**Q7.** Assuming no upsets, which player beat seed No. 25?

(a) Seed 8

(b) Seed 6

(c) Seed 1

(d) Seed 14

**Q8.** If the player seeded 13 won the tournament, then what is minimum number of upsets in the tournament?

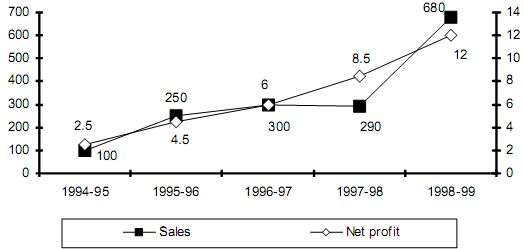
(a) 2

(b) 3

(c) 4

(d) 5

**Directions for Questions (Q9- Q12):** The figure below represents sales and net profit in Rs. crore of IVP Ltd. for five years from 1994-95 to1998-99. During this period the sales increased from Rs. 100 crore to Rs. 680 crore. Correspondingly, the net profit increased from Rs. 2.5 crore to Rs. 12 crore. Net profit is defined as the excess of sales over total costs.



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**Q9.** The highest percentage of growth in sales, relative to the previous year, occurred in

(a) 1995-96

(b) 1996-97

(c) 1997-98

(d) 1998-99

**Q10.** The highest percentage growth in net profit, relative to the previous year, was achieved in

(a) 1998-99

(b) 1997-98

(c) 1996-97

(d) 1995-96

**Q11.** Defining profitability as the ratio of net profit to sales, IVP Ltd., recorded the highest profitability in

(a) 1998-99

(b) 1997-98

(c) 1994-95

(d) 1996-97

**Q12.** With profitability as defined in question 3, it can be concluded that

(a) Profitability is non-decreasing during the 5 years fm 1994-95 to1998-99

(b) Profitability is non-increasing during the 5 years fm 1994-95 to1998-99.

(c) Profitability remained constant during the 5 years fm 1994-95 to1998-99. (d) None of the above

**Directions for Questions (Q13- Q17):**

Study the pie-charts and answer the questions below them.

|  |
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**13.** How much extra is the availability of spending on other items, during 1980-2005 periods?

(a) Rs 2760

(b) Rs 2000

(c) Rs 3150

(d) Rs 2250

(e) Rs 2350

**Q14.** The ratio of actual spending on food in 2005 to that in 1980 is:

(a) 10 : 7

(b) 20: 9

(c) 5: 2

(d) 3: 1

(e) 2: 3

**15**. In 1980, people in rural India had 30% less earning and spent 40% on food and out of this, expenditure on milk products was only 10%. Its value is:

(a) Rs 84

(b) Rs 92

(c) Rs 60

(d) Rs 42

(e) Rs 90

**Q16.** How much other food items are available in 2005, in Rs terms?

(a) Rs 180

(b) Rs 93

(c) Rs 642

(d) Rs 1200

(e) Rs 558

**Q17**. In relative per cent terms, is there an increase or decrease in the expenditure on milk etc?

(a) Decrease

(b) Increase

(c) Neither

(d) Can’t be said

(e) No change

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| 7.6 Additional Exercise from various competitive examinations |

**Directions for Questions (Q18-Q22):** The following bar chart gives the amount of imports and exports of a leather company over the years (Bars without lines represents Imports and Bars with lines represents Exports) in Rupees crores:

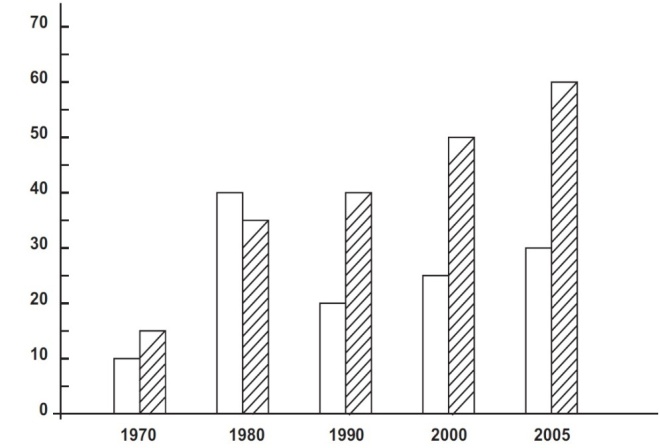
**Q18.** Total imports for Ist2 years are what per cent of total exports for these 2 years?

(a) 50%

(b) 100%

(c) 120%

(d) 80%



**Q19.** For which years is the value of the total imports equal to the total exports?

(a) 1970, 1990

(b) 1970, 1980

(c) 1990, 2000

(d) 2000, 2005

**Q20.** The ratio of exports to imports was maximum in the year:

(a) 1990, 2000, 2005

(b) 1900, 2005

(c) 2000, 2005

(d) 1970, 1990, 2000

**Q21.** When was the per cent increase in exports, over the previous 10-year, the least?

(a) 1980

(b) 2000

(c) 1990

(d) 2005

**Q22.** The ratio of total exports to the total of imports for all the years is: (a) 8 : 7

(b) 6 : 5

(c) 4 : 3

(d) 8 : 5

**Direction for Questions (Q23 to Q26):**

The following table shows the break-up of actual costs incurred by a company in last five years (year 2002 to year 2006) to produce a particular product. The production capacity of the company is 2000 units. The selling price for the year 2006 was Rs. 125 per unit. Some costs change almost in direct proportion to the change in volume of production, while others do not follow any obvious pattern of change with respect to the volume of production and hence are considered fixed. Using the information provided for the year 2006 as the basis for projecting the figures for the year 2007, answer the following questions.



**Q23.** What is the approximate cost per unit in rupees, if the company produces and sells 1400 units in the year 2007? (a) 104

(b) 107

(c) 110

(d) 115

(e) 116

**Q24.** What is the minimum number of units that the company needs to produce and sell to avoid any loss?

(a) 313

(b) 350

(c) 384

(d) 747

(e) 928

**Q25.** If the company reduces the price by 5%, it can produce and sell as many units as it desires. How many units should the company produce to maximize its profit? (a) 1400

(b) 1600

(c) 1800

(d) 1900

(e) 2000

**Q26**. Given that the company cannot sell more than 1700 units, and it will have to reduce the price by Rs. 5 for all units, if it wants to sell more than 1400 units, what is the maximum profit, in rupees, that the company can earn?

(a) 25,400

(b) 24,400

(c) 31,400

(d) 32,900

(e) 32,000

**Direction for Questions (Q27-Q30):** These questions are based on the following information.

Each of the four persons – A, B, C and D – bought a special item – X, at different costs among Rs. 150, Rs. 225, Rs. 300 and Rs. 375 and sold them at different percentages of profit among 30%, 40%, 50% and 60% not necessarily in that order. It is also known that:

(i) The profit of B is less than that of D and is more than that of A, whose profit is more than that of C.

(ii) Neither the cost price is the highest nor is the profit percentage the highest for the person whose profit is the highest.

(iii) The person whose profit is Rs. 112.50 is not the one whose profit is the highest. (iv) No two persons obtained the same amount of profit.

**Q27.** What is the highest profit? (a) Rs. 180

(b) Rs. 150

(c) Rs. 135

(d) Rs. 112.50

**Q28.** What is the lowest profit? (a) Rs. 60

(b) Rs. 45

(c) Rs. 75

(d) Rs. 67.50

**Q29.** What is the cost price of item – X for B?

(a) Rs. 300

(b) Rs. 375

(c) Rs. 225

(d) None of these

**Q30.** For whom is the percentage of profit the minimum? (a) D

(b) B

(c) C

(d) A

**Chapter 8**

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| --- |
| LOGARITHMS |

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| 8.1 Learning objective: |

To learn about the Properties and Applications of Logarithms

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| 8.2 Introduction: |

**Logarithm:**

If a is a positive real number, other than 1 and am = x, then we write: **m = logax** and we say that the value of log x to the base a is m.

Examples:

(i). 103 1000   ⇒   log10 1000 = 3.

(ii) 34 = 81   ⇒   log3 81 = 4.

(iii)

(iv) (.1)2 = .01   ⇒ log(.1) .01 = 2.

**Properties of Logarithms:**

1. loga (xy) = loga x + loga y

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 2. loga |  | x |  | = loga x - loga y |
| y |

3. logx x = 1

4. loga 1 = 0

5. loga (xn) = n(loga x)

|  |  |
| --- | --- |
| 6. loga x = | 1 |
| logx a |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 7. loga x = | logb x | = | log x |  |
| logb a | log a |

**Common Logarithms:**

Logarithms to the base 10 are known as

common logarithms.

The logarithm of a number contains two parts, namely 'characteristic' and 'mantissa'.

**Characteristic:** The internal part of the logarithm of a number is called its **characteristic.**

**Case I:** When the number is greater than 1.In this case, the characteristic is one less than the number of digits in the left of the decimal point in the given number.

**Case II:** When the number is less than 1.

In this case, the characteristic is one more than the number of zeros between the decimal point and the first significant digit of the number and it is negative.

Instead of -1, -2 etc. we write 1 (one bar), 2 (two bar), etc.

**Examples:**

|  |  |  |  |
| --- | --- | --- | --- |
| Number | Characteristic | Number | Characteristic |
| 654.24 | 2 | 0.6453 | 1 |
| 26.649 | 1 | 0.06134 | 2 |
| 8.3547 | 0 | 0.00123 | 3 |

**Mantissa:** The decimal part of the logarithm of a number is known is its **mantissa.** For mantissa, we look through log table.

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| 8.3 Worked out examples |

**Q1**. If logx 4 = 1/4, then x is equal to

**Explanatory Answer:**

LogX4 = ¼

⇒x1/4=4

x=44

x = 256.

**Q2**. If log 2 = 0.3010 and log 3 = 0.4771, the value of log5 512 is:

**Explanatory Answer:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| log5 512 | |  |  | | --- | --- | | = | log 512 | | log 5 | |
|  | |  |  | | --- | --- | | = | log 29 | | log (10/2) | |
|  | |  |  | | --- | --- | | = | 9 log 2 | | log 10 - log 2 | |
|  | |  |  | | --- | --- | | = | (9 x 0.3010) | | 1 - 0.3010 | |
|  | |  |  | | --- | --- | | = | 2.709 | | 0.699 | |
|  | |  |  | | --- | --- | | = | 2709 | | 699 | |
|  | = 3.876 |

**Q3**. The value of ()

is:

**Explanatory Answer:**

Given expression is log603 + log604 + log605

= log60(3

= log6060 = 1.

**Q4**. The value of (log3 4) (log4 5) (log5 6) (log6 7) (log7 8) (log8 9) is :

**Explanatory Answer:**

Given expression =

= = = 2

**Q5.** If log 3 = 0.477 and (1000)x= 3, then x equals :

**Explanatory Answer:**

(1000)x = 3

log(1000)x = log 3

x log 1000 = log 3

x log (103) = log 3

3x log 10 = log 3

3x = log 3

x = 0.477/3 = 0.159

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| 8.4 Class Work Problems |

**Q1.** The value of log 3437 is:

(a) 1/3

(b) -3

(c) -1/3

(d) 3

**Q2.** The value of log√232 is:

(a) 5/2

(b) 5

(c) 10

(d) 1/10

**Q3.** The value of log(.01)(1000) is:

(a) 1/3

(b) -1/3

(c) 3/2

(d) -3/2

**Q4.** If logx(9/16) = -1/2, then x is equal to:

(a) -3/4

(b) ¾

(c) 81/256

(d) 256/81

**Q5.** If logxy = 100 and log 2x =10, then the value of y =?

(a) 210

(b) 2100

(c) 21000

(d) 210000

**Q6.** Which of the following statements are not correct?

(a) log 1010 = 1

(b) log(2 + 3) = log(2 × 3)

(c) log 101 = 0

(d) log(1 + 2 + 3) = log1 + log2 + log3

**Q7.** The value of log2(log5625) is:

(a) 2

(b) 5

(c) 10

(d) 15

**Q8.** If ax =by, then :

(a) log(a/b) = x/y

(b) log a/log b = x/y

(c) log a/log b = y/x

(d) None of these

**Q9.** 2log105 + log108 – ½ log 104 = ?

(a) 2

(b) 4

(c) 2 + 2 log102

(d) 4 – 4 log102

**Q10.** If loga(ab) = x, then logb(ab) is:

(a) 1/x

(b)

(c)

(d)

**Q11**. (log53) × (log 3625) =?

(a) 1

(b) 2

(c) 3

(d) 4

**Q12.** If log1227 = a, then log616 is:

(a)

(b)

(c)

(d)

**Q13.** If log (a/b) + log(b/a) = log (a+b), then:

(a) a+b = 1

(b) a –b = 1

(c) a=b

(d) a2-b2 = 1

**Q14.** logba×logcb×logac =?

(a) 0

(b) 1

(c) abc

(d) a+b+c

**Q15.** [log(a2/bc) + log(b2/ac) + log(c2/ab)] = ?

(a) 0

(b) 1

(c) 2

(d) abc

**Q16.** If a=bx , b=cy and c=az, then the value of xyz =?

(a) -1

(b) 0

(c) 1

(d) abc

**Q17.** If log 27 = 1.431, find the value of log 9?

(a) 0.934

(b) 0.945

(c) 0.954

(d) 0.958

**Q18.** If log105 + log10(5x+1) = log10(x+5) +1, then the value of x is:

(a) 1

(b) 3

(c) 5

(d) 10

**Q19.** The value of 16log45 is:

(a) 5/64

(b) 5

(c) 16

(d) 25

**Q20.** If log102 = 0.3010, the value of log1080 is:

(a) 1.6020

(b) 1.9030

(c) 3.9030

(d) None of these

**Q21.** If log 2 =0.30103, the number of digits in 264is:

(a) 18

(b) 19

(c) 20

(d) 21

**Q22.** If log 2 =0.30103, then the number of digits in 520 is:

(a) 14

(b) 16

(c) 18

(d) 25

**Q23**.If log102 = 0.3010 and log103 = 0.4771, then the value of log5512 is:

(a) 2.870

(b) 2.967

(c) 3.876

(d) 3.912

**Q24.** If log55 log49 log32 is equal to:

(a) 1

(b) 3/2

(c) 2

(d) 5

**Q25**. log 360 is equal to:

(a) 2 log2 + 3 log3

(b) 3 log2 +2 log3

(c) 3 log2+2 log3 – log5

(d) 3 log2 +2 log3 +log 5

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| 8.5 Additional Exercise from various competitive examinations |

**Q1.** If log 2=x, log 3=y and log 7=z, then the value of is:

(a)

(b)

(c)

(d)

**Q2.** If log(0.57)=1.756, then the value of

is:

(a) 0.902

(b) 1.902

(c) 1.146

(d) 2.146

**Q3.** If log 90=1.9542 then log 3 equals to

(a) 0.9771

(b) 0.6514

(c) 0.4771

(d) 0.3181

**Q4.** If log102 =0.3010 and log107=0.8451 , then the value of log10 2.8 is:

(a) 0.4771

(b) 1.4471

(c) 2.4471

(d) 14.471

**Q5.** The value of is:

(a) log 2

(b) 2 log 2

(c) log 3

(d) log 5

**Q6.** If log 3=0.477 and (1000)x=3 , then x equals to

(a) 0.159

(b) 10

(c) 0.0477

(d) 0.0159

**Q7.** If then

(a)

(b)

(c)

(d)

**Q8.** If log 10 x – log 10 = 2 log x 10, then the possible value of x is given by

(a) 10

(b)

(c)

(d) None of these

**Q9.** What is the sum of n terms in the series:

log m+ log(m2 /n) + log(m3/n2 )+ log(m4/n3 )+……

**Self Study**

**Chapter 9**

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| Symbol Based Problems |

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| 9.1 Home Assessment Problems |

**Directions for Questions (Q1-Q5):**

α means greater than

β means equal to

θ means not less than

γ means less than

* means not equal to
* means not greater than

Which of the following could be a correct or proper inference given the following

**Q1.** a α2b and 2bθ r

(a) a αr

(b) aβ r

(c) aγr

(d) aηr

(e) None of these

**Q2**. pη 3q and 3q β2r

(a)pα3r

(b) pβ2r

(c) pθ2r

(d) pγ3r

(e) None of these

**Q3.** 2xη y and xα o

(a) 2xγo

(b) yβo

(c) oγy

(d) yβo

(e) None of these

**Q4**. 3x θ2y and 2yβ 3z

(a) 3xβz

(b) 2yδ 3x

(c) xδz

(d) 2yθ3x

(e) 2xθ3y

**Q5.** 2x δy and yβ 3z

(a) 3zη3y

(b) yδ6x

(c) 2xη3z

(d) 2xδ 3z

(e) None of these

**Directions for Questions (Q6-Q10):** In these questions some symbols have been used for some mathematical operations as indicated below. Find the correct answer in each question.

× for ‘greater than’

for ‘not less than’

÷ for ‘not equal to’

φ for ‘equal to’

+ for ‘ not greater than’

Δ for ‘less than’

**Q6.** If a b× c, it implies that (a) a× c+ b

(b) c×b×a

(c) aΔbc

(d) abφc

**Q7**. If a× bΔ c, it follows that (a) aφcΔb

(b) ba×c

(c) ab+c

(d) c+ba

**Q8.** If a+ b÷ c, it does not imply that (a) aΔb×c

(b) bΔc×a

(c) a×cΔb

(d) b×aφc

**Q9.** If aΔ bΔ c, it does not imply that

(a) b×aΔc

(b) c×b×a

(c) aΔc×b

(d) a×c×b

**Q10**. If a+ b+ c, it does not imply that (a) aφbΔc

(b) cφbΔa

(c) aΔbΔc

(d) aΔbφc

**Directions for Questions (Q11-Q15):**

Identify one amongst the four responses which would be a correct inference that can be drawn from given definitions and premises.

**Definitions**: ‘A’ means greater than, ‘B’ means equal to, ‘C’ means not less than, ‘D’ means less than, ‘E’ means not equal to and ‘F’ means not greater than.

**Q11. Premises:** (a A 2b) and (2b C r) (a) a B r

(b) a F r

(c) a A r

(d) a D r

**Q12. Premises:** (x F y) and (X A o) (a) y D o

(b) y A o

(c) y F o

(d) y B o

**Q13. Premises:** (2x E y) and (2y F 3z) (a) 2x F 3y

(b) y B 6x

(c) y D 3z

(d) 3z B 3y

**Q14. Premises:** (2a C 3b) and (3b B c) (a) a C c

(b) 2a D c

(c) 2a C c

(d) 2a B 3c

**Q15.Premises:** (p F 3q) and (3q D 2r) (a) p D 2r

(b) p A 2r

(c) p C 2r

(d) p B 2r

**Directions for Questions (Q16-Q20):**

> stands for =

< stands for ≠

× stands for >

+ stands for <

= stands for  ~~>~~ (not greater than)

- stands for ~~<~~ (not less than)

**Q16.** If α× β>γ , it implies that (a) α- β = γ

(b) α× β× γ

(c) α+ β× γ

(d) α- β > γ

(e) None of these

**Q17.** If α- β× γ , it implies that

(a) α> β < γ

(b) α× β> γ

(c) α+ β× γ

(d) α< β× γ

(e) None of these

**Q18.** If α> β× γ , it implies that

(a) α- β> γ

(b) α< β- γ

(c) α= β= γ

(d) α= β> γ

(e) α- β- γ

**Q19.** If α× β- γ , it implies that

(a) α= β× γ

(b) α- β+ γ

(c) α< β+ γ

(d) α <β× γ

(e) None of these

**Q20.** If α+ β×γ , it implies that

(a) α= β= γ

(b) α< β= γ

(c) α= β- γ

(d) α× β< γ

(e) α> β + γ

**Q21.** If → stands for subtraction; ⏐stands for addition; ↔ stands for multiply; ↓ stands for divide; ⎯ stands for equal to;← stands for greater than, then which of the following alternatives are true?

(a) 4⏐6↔2←3→12⏐12

(b) 10↓5↔5⎯9→3⏐4

(c) 15↔2→5⎯12↓4⏐3

(d) 13↓13⏐1←20→5↔2

**Q22.** If + means ×, - means + and ÷, find the value of 5+4-18×3

**Q23.** If ‘P’ denotes ‘÷’, ‘Q’ denotes ‘×’, ‘R’ denotes ‘÷’ and ‘S’ denotes ‘-’ , then 18 Q 12 P 4 R 5 S 6 = ?

**Directions for Questions (Q24&Q25):**

If ‘-‘ stands for addition, ‘+’ for multiplication, ‘÷’ for subtraction and ‘×’ for division, then compute

**Q24.**5+2-12×6÷2

**Q25.**12×12-25 ÷2

**Directions for Questions (Q26-30):**

if the given interchanges are made in signs and numbers, which one of the four equations would be correct?

**Q26.**Given interchanges: Sign + and ÷ and numbers 2 and 4.

(a) 2 + 4 ÷ 3 = 3.0

(b)4 + 2 ÷ 6 = 1.5

(c) 4 ÷ 2 + 3 = 4.0

(d) 2 + 4 ÷ 6 = 8.0

**Q27.** Given interchanges: Sign - and ÷ and numbers 4 and 8.

(a) 6-8÷4=1.0

(b) 8-6÷4=1.0

(c) 4÷8-2=6.0

(d) 4-8÷6=2.0

**Q28.** Given interchanges: Sign + and × and numbers 4 and 5.

(a) 5×4÷20=40

(b) 5×4÷20=85

(c) 5×4÷20=104

(d) 5×4÷20=185

**Q29.** Given interchanges: Sign + and - and numbers 4 and 8.

(a) 4÷8-12=16

(b) 4-8+12=0

(c) 8÷4-12=24

(d) 8-4÷12=8

**Q30.** Given interchanges: Sign + and ÷ and numbers 3 and 6.

(a) 6-3×2=9

(b) 3-6×8=10

(c) 6×3-4=15

(d) 3×6-4=33

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| PART II |

**Chapter 1**

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| Sequence and Series. |

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| 1.1 Learning objective: |

To learn about Arithmetic progression, Geometric progression, Harmonic progression and Arithmetic geometric progression.

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| 1.2 Introduction: |

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| **1.2.1 Arithmetic Progression (AP)** |

The progression of the form: a, a + d, a + 2d, a + 3d … is known as an AP with first term = a, and common difference = d.  
  
In an AP a, a + d, a + 2d, a + 3d, …, we have:  
  
(i) nth term,

Tn = a + (n – 1) d

(ii) Sum to n terms,

where l is the last term.

(iii) If a, b, c are in AP, then b is called with arithmetic mean (AM) between a and c. In this case,

(iv) If a, a1, a2 … an, b are in AP we say that a1, a2 … an are the n arithmetic means between a and b.

(v)   It is convenient to take: three numbers in AP as (a – d), a, (a + d)

Four numbers in AP as

(a – 3d), (a – d), (a + d), (a + 3d)

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| **1.2.2 Geometric Progression (GP)** |

The progression of the form:

a, ar, ar2, ar3, … is known as a GP with first term = a and common ratio = r

(i) nth term, Tn = arn– 1

(ii) Sum to n terms,

, When < 1 and

, When > 1

(iii) If a, b, c are in GP, then b is the geometric mean (GM) between a and c.

In this case, .

(iv) If a, a1, a2 … an, b are in GP we say that a1, a2 … an are n geometric means between a and b.

(v) The sum of an infinite GP a, ar, ar2… is when < 1

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| **1.2.3 Harmonic Progression (HP)** |

The progression a1, a2, a3… is called an HP, if ,is an AP.

If a, b, c are in HP, then b is the harmonic mean (HM) between a and c.

In this case,

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| **1.2.4 Relationship between the Means of AP , GP and HP** |

If AM, GM and HM be the arithmetic, geometric and harmonic means between two positive numbers a and b, then the following results hold:

GM2 = AM x HM

AM ≥ GM ≥ HM.

Can AM=GM=HM? Explain

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| 1.3 Worked out examples |

**Q1.** What is the sum of all 3 digit numbers that leave a remainder of '2' when divided by 3?

(a) 300

(b) 164,850

(c) 164,751

(d) 166,751

(e) 166,850

The correct choice is (b) and the correct answer is 164,850.

**Explanatory Answer**

101 leaves a remainder 2 as 101= 3x33+2

998 leaves a remainder 2 as 998 = 332x3+2

Therefore the required sum must be 101+104+107+…+998

There are totally 332-33+1 = 300 such numbers. Therefore n= 300

Sum of an AP =

Can you solve this without using formula?

**Q2**. **200 logs are stacked in the following manner:  20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?**

(a) 14

(b) 15

(c) 16

(d) 20

The correct choice is (c) and the correct answer is 16.

**Explanatory Answer:**

The series is 20+19+18+…

a= 20, d= -1

and n < 20

n=16

Can you solve this by any other method?

**Q3.** Given A = 265 and

B = (264+263+262+...+20)

(a) B is 264 larger than A

(b) A and B are equal

(c) B is larger than A by 1

(d) A is larger than B by 1

Correct Answer is A is larger than B by 1. Correct Choice is (d)

**Explanatory Answer:**

B = 264+263+262+...+20

= 20+21+22+23+...+264

See a pattern.

20=1=21-1

20+21=3=22-1

20+21+22=7=23-1

.

.

.

20+21+22+23+...+264=265-1

B= A-1.

Can you solve this using formula?

**Q4.** The first term of an arithmetic progression is the common ratio of a GP and the first term of the same GP is the common difference of the AP. The sum of the first 10 terms of the AP is 155 and the sum of the first two terms of the GP is 9.

Find the first term of the AP?  
(a) 2 or 3

(b) 25/2 or 3

(c) 2 or 25/2

(d) 1 or 2

Correct Answer is (c)

**Explanatory Answer:**

Let a, a + d. . . .be the (A) P   
and d, da, da2 . . .. be the G.P

and d(1+a) = 9

Solving the above two we get r = 25/2 or 2  
Is there any other way to solve this problem?

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| 1.4 Class Work Problems |

**Q1.** If a, b, c are the pth, qth and rth term of an A.P, then the value of a (q-r) +b(r-p) +c (p-q) is \_\_\_\_\_

(a) 0

(b) 1

(c) -1

(d) None of these

**Q2.** If pth term of an A.P is q and the qth term is p the value of the rth term is \_\_\_.

(a) p-q-r

(b) p+q-r

(c) p+q+r

(d) r-p-q

**Q3.** Find the sum to n terms of ****

(a)

(b)

(c) (n-1)

(d) (n+1)

**Q4.** If Sn the sum of first n terms in a series is given by 2n2+3n the series is in \_\_\_.

(a) A.P

(b) G.P

(c) H.P

(d) None.

**Q5.** If a, b, c be the sums of p, q, r terms respectively of an A.P the value of

 is\_\_\_\_.

(a) 0

(b) 1

(c) -1

(d) None

**Q6.** The number x, 8, y are in G.P and the numbers x, y, -8 are in A.P The value of x, y are \_\_\_\_.

(a) 16, 4

(b) 4, 16

(c) Both (a) and (b)

(d) None

**Q7.** If a, b, c are the pth , qth and rth terms of a G.P the value of a q-r.b r-p. c p-q is \_\_\_\_

(a) 0

(b) 1

(c) -1

(d) None

**Q8.** If a, b, c, d are in G.P then the value of b(ab-cd) -(c+a) (b2-c2) is \_\_\_\_

(a) 0

(b) 1

(c) -1

(d) None

**Q9.** The sum up to infinity of the series

(1+2-2) +(2-1+2-4) +(2-2+2-6) +…is

(a)

(b)

(c)

(d) None

**Q10.** Geometric mean between -2 and -8 is

(a) ±4

(b) 4

(c) -4

(d) Here G.M is meaningless and so it does not exist.

**Q11.** If A.M, G.M, H.M represents arithmetic mean, Geometric mean and harmonic mean respectively and if they exist, then

(a) A.M ≤ G.M ≤ H.M

(b) A.M ≥ G.M ≥ H.M

(c) it depends on the data

(d) None

**Q12.** Consider two positive numbers ‘a’ and ‘b’. If A.M, G.M, H.M represents arithmetic mean, Geometric mean and harmonic mean respectively, then A.M, G.M,H.M are in

(a) Arithmetic progression (A.P)

(b) Geometric progression (G.P)

(c) Harmonic progression (H.P)

(d) None

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| 1.5 Home assignment |

**Q1.** How many terms of the series 3, 6, 12, 24, … make a sum of 1533?

**Q2.** Find the common ratio of G.P., whose first term is 3, last term is 3072 and the sum of the series is 4095.

**Q3.** In an infinite G.P., each term is equal to four times the sum of the terms that follow. Find the common ratio.

**Q4.** 3+33+333+…+ up to n terms is \_\_\_\_.

**Q5.** 0.6+ 0.66 +0.666+ … + up to n terms is \_\_\_\_.

**Q6.** Insert 4 G.M’s between 3 and 729.

**Q7.** The G.M between -2 and -8 is \_\_\_\_\_.

**Q8.** If, are in H.P., such that abc = 9, then the minimum value of b is \_\_\_\_.

**Q9.** The sum of the infinite series

1 + (1 +b) r = (1+b+b2) r2+… where ‘r’ and ‘b’ are proper fractions is \_\_\_\_\_\_\_.

**Q10.** 1 + + ++… = \_\_\_\_\_\_\_.

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| 1.6 Additional problems |

**Q1.** The 5th term of an A.P is 15 and the 9th term is 23. Find the 14th term.

**Q2.** If 9 times the 9th term of an A.P is equal to 15 times the 15th term in the A.P., what is the 24th term?

**Q3.** Find the number of terms and sum of the series 24, 21, 18, …,3.

**Q4.** Find the sum of an A.P of 14 terms and whose first and the last terms are 8 and 99 respectively.

**Q5.** The sum of 5 terms in an A.P is 60 and the product of extreme terms is 128. Then find the largest term.

**Q6.** The sum of k terms of an A.P is 2k2+4k. Then the nth term is \_\_\_\_\_.

**Q7.** If a, b, c are in A.P., then are in \_\_\_\_.

**Q8.** If a, b, c are in A.P., then b + c, c + a, a + b are in \_\_\_\_\_.

**Q9.** The sum of three numbers in A. P is 9 and the sum of their squares is 59. The least of the numbers is \_\_\_\_\_\_.

**Q10.** If eleven times the eleventh term of an A.P is equal to seven times the seventh term, the eighteenth term of the A.P is \_\_\_\_\_.

**Q11.** Insert 5 A.M’s between 3 and 27.

**Q12.** If the nth term of an A.P is ‘p’ and the mth term of the same A.P is ‘q’, then (m + n) th term of the same A.P is \_\_\_\_.

**Q13.** If nth terms of two series 43 + 45 + 47 + 49 +… and 5 + 8 + 11 + … are equal, then the value of n is \_\_\_\_.

**Q14.** The H.M between two numbers is 64/17 and the G.M is 8. The numbers are \_\_\_\_\_.

**Q15.** If is the A.M between ‘a’ and ‘b’, then ‘n’ is equal to \_\_\_.

**Q16.** The sum to infinity of

1+3x+6x2+10x3+…, where -1< x < 1 is \_\_\_\_\_.

**Q17.** The reciprocals of the terms of a G.P form a/an \_\_\_\_\_.

**Q18.** Find the sum to infinite terms of a G.P whose fourth term is 4/49 and first term is 28.

**Q19.** The 5th and 9th terms of a G.P are 2 and 1/8. Find the 15th term of the series.

**Q20.** The sum of three numbers which are in G.P. is 21. If their product is 216, find the three numbers.

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| 1.7 Self assessment 1 |

**Q1.** If a, b, c, d are in Harmonic progression and (a-c)(b-d) =K (a-b)(c-d), then K = ?

(a) 4

(b) 2

(c) 3

(d) 1

(e) 5

**Q2.** If a + b, b +c, c + a are in H.P. show that a2,b2,c2 are in …..

(a) Geometric Progression

(b) Arithmetic Progression

(c) Harmonic Progression

(d) They form a “mixed” series, but it is not an AP, GP or HP.

(e) No particular progression or

sequence.

**Q3.** If x, y, z are in A.P. x, x y, z are in G.P then x, x2y, z are in …

(a) Arithmetic Progression

(b) Geometric Progression

(c) No specific series or progression

(d) For certain ratios of x/z, in Harmonic Progression

(e) Always in Harmonic Progression

**Q4.** If a, b, c are in H.P then prove that

are in ….

(a) Arithmetic Progression

(b) Geometric Progression

(c) No particular sequence

(d) Harmonic Progression

**Q5.** If the AM of a set of two observations is 9 and its GM is 6. then the HM of the set of observations is

(a) 4.5

(b) 36

(c) 12

(d) 4

(e) 1.5

**Q6.** If a, b, c, d are in HP, then

ab + bc + cd = ?

(a) 3ad

(b) 2ad

(c) ad

(d) 1

**Q7.** If a, b, c and d are in HP then

(a) ab>cd

(b) ac>bd

(c) ad>bc

(d) None of the above

**Q8.** If , y, are in HP then x, y, z are in

(a) AP

(b) GP

(c) HP

(d) None of the above

**Q9.** Five numbers a, b, c, d and e are such that a, b and c are in AP; b, c and d are in GP and c, d and e are in HP. If a = 2, e = 18; then what are the values of b, c and d?

(a) 2, 6, 18

(b) 4, 6, 9

(c) 4, 6, 8

(d) -2,-6, 18

# Q10. Find three numbers in geometric progression whose sum is 21 and sum of their squares is 189?

(a) 4, 5, 7

(b) 4, 6, 7

(c) 3, 6, 9

(d) 3, 6, 12

(e) 3, 7,11

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| 1.8 Self assessment 2 |

**Q1.** What is the value of the following expression where all the whole numbers 1 through 99 are included and the repeated operations are add, add, subtract?   
0 + 1 + 2 – 3 + 4 + 5 – 6 + 7 + 8 – 9 + . . . + 97 + 98 – 99

**Q2.** Complete the ???? row in this problem. Explain how you found the solution.

1   
11   
21   
1211   
111221   
312211   
13112221   
1113213211   
??????????????

**Q3.** In the following decimal, how many 2's are there in all before the hundredth 3?

0.23223222322223. . . .

**Q4.** How many integers from 10 to 40 can be written as the sum of 2 consecutive integers?

How many integers from 10 to 40 can be written as the sum of 3 consecutive integers?

How many integers from 10 to 40 can be written as the sum of 4 consecutive integers?

Can you find a pattern? Explain.

**Q5.** Hansel has goldfish that quadruple, or become four times as many, every month. Gretel has goldfish that increase by 20 every month. Right now, Hansel has 4 goldfish and Gretel has 128 goldfish. In how many months will they have the same number of goldfish? Show how you arrived at your answer.

**Q6.** Nine stones are arranged in a straight line. They are counted from left to right as 1,2,3,..., 9 and then from right to left so that the stone previously counted as 8 is counted as 10. The pattern is continued to the left until the stone previously counted as 1 is counted as 17. The pattern then reverses so that the stone originally counted as 2 (then 16) is counted as 18, 3 as 19, and so on. The counting continues in this way. Which of the original stones is counted as 599? Express your answer as the first number assigned to that stone.

**Q7.** How many terms are in the sequence 3, 5, 9, 15, 23, ... , 423 ?

**Q8**. If a piece of paper could be folded in half fifty times, what would be the thickness of the folded paper? A ream (500 sheets) of paper is 2 inches thick.

**Q9.** Does a natural number n exist such that 1+2+3+4+...+n is a three-digit number with identical digits?

**Q10.** Write the rational number

5.231313131… as the ratio of two

integers.

**Chapter 2**

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| TIME, SPEED AND DISTANCE |

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| 2.1 Learning objective: |

In this chapter, time speed and distance, relative and resultant speed will be discussed.

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| 2.2 Introduction: |

Questions from Time, Speed and Distance can be classified into 4 basic categories.

1. Problems based on Trains
2. Problems based on Boats and Streams
3. Problems based on Races and Games
4. Miscellaneous type questions

In order to answer the questions from this section the basic equation that is used will be :

Speed =

Also, one need to be familiar with the concepts related to Ratio, Proportion and Variations.

Following basic conversion concept will be also really useful for solving the questions:

* 1 Km/Hr = (5/18) m/sec
* 1 m/sec = (18/5) Km/Hr

**Average Speed**

Consider a situation where a person travels from point A to point B at a speed of x m/s. He immediately makes a return trip to B from A at a speed of y m/s. Now the average speed of the journey is given by:

Average Speed =

Average Speed =

In solving problems on trains, the following points should be kept in mind.

Time taken by a train to cross a pole (or Signal post or, a standing man)

= Length of the train/Speed of the train

Time taken by a train to cross a bridge (or tunnel or a train at rest or a platform)

= (Length of the train+ Length of the bridge) /Speed of the train

Consider a train of length L1 moving at a speed of V1, trying to cross another moving object of length L2 travelling at a speed of V2.

If the object is moving in the opposite direction as that of the train, time taken by the train to cross the moving object will be

If the object is moving in the same direction as that of the train, time taken by the train to cross the moving object will be

*(We will be taking the absolute value of the denominator in this case)*

For solving questions based on Boats and Streams, following equations are used.

1. If the speed of a boat/man in still water be x km/hr and that of stream be y km/hr

1. Speed of the boat/man downstream = (x+y) km/hr
2. Speed of the boat/man upstream = (x-y) km/hr

2. If the speed of a boat downstream is u km/hr and speed upstream is v km/hr

a) Speed of the boat in still water =

b) Speed of the current =

**For solving questions based on races/games one need to be familiar with the interpretation aspects of certain statements.**

Consider the statement: In a 100 metres race, P beats Q by 10 metres or 2 seconds. Assume that P took 15 Seconds to cover this distance.This statement means – when P completed 100 metres, Q had covered only (100-10) = 90 metres. Also, when P completed the race, Q was 2 seconds away from the finishing point. It means, Q will be covering the remaining 10 metres in 2 seconds.

**For solving questions based on people/objects moving along circular tracks:**

Consider a circular track of length L metres along which two people- P and Q are running at a speed of x m/s and y m/s. They both started running simultaneously from the same point. P will take seconds to complete one round of running. Similarly, Q will take seconds to complete one round of running. So, for P and Q in order to meet at the point from where they started the race for the first time, we need to take the LCM of and

If P and Q are running in the same direction (both clockwise or counter - clockwise) where P is travelling at a faster pace, then their relative speed will be (x-y) m/sec. So, the time taken by P to meet Q for the first time will be

If P and Q are running in opposite direction (one clockwise and the other counter-clockwise) where P is travelling at a faster pace, then their relative speed will be (x+y) m/sec. So, the time taken by P to meet Q for the first time will be

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| **2.3 Quiz** |

**Q1**. Bunti travels at a speed of 20 Kmph. After 5 hours, Bubli starts from the same point and travels towards Bunti, at a speed of 40 Kmph. What is the distance Bunti would have run, before Bubli catches up with him?

(a) 200

(b) 225

(c) 250

(d) 300

**Q2**. To cover a distance of 60 Kms, Ram took 4 hours more than Shyam. If Ram doubles his speed, he would take 2 hours less than Shyam. What is Ram’s speed in Kmph?

(a) 5

(b) 7.5

(c) 6

(d) 6.5

**Q3.** A, B, C and D decides to run a 1800 metres race. A beats B by 360 metres; B beats C by 225 metres; D beats C by 400 m. Which of the following conclusions is true?

(a) A beats D by 200 metres

(b) A beats D by 180 metres

(c) D beats A by 200 metres

(d) D beats A by 180 metres

**Q4**. In still water, a man takes 16 seconds to travel 256 metres by boat. He is crossing a river of width 550 metres in the minimum possible time using a boat travelling at a speed of 20 m/sec. What is the speed of the river in m/second?

(a) 14

(b) 12

(c) 15

(d) Can’t Say

**Q5**. P is 2 times as fast as Q. In a race P beats Q by 120 metres. What is the length of the race?

(a) 100m

(b) 200m

(c) 250m

(d) 350m

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| 2.4 Worked out examples |

**Q1**. A car covers a certain distance by traveling at 20 kmph for the first three hours and at 30 kmph for the next three hours. What is the total distance covered by the car?

(a) 155 km

(b) 180 km

(c) 150 km

(d) 175 km

Correct option is (c)

**Explanatory Answer:**

Distance = Speed X Time

Total Distance covered by the car = 20 X 3 + 30 X 3 = 150 Km

**Q2.** Two towns A and B are separated by 60 km. Two friends started at the same time - one from A to B and the other from B to A at speeds of 6 kmph and 9 kmph respectively. Where do the two cross each other?

(a) 20 km from A

(b) 24 km from A

(c) 26 km from A

(d) 32 km from A

Correct option is (b)

**Explanatory Answer:**

The time taken by the two friends to cross each other = (60/15) = 4 hours

So, the crossing will happen at 6 X 4 = 24 Kms from A or 9 X 4 = 36 Kms from B

**Q3**. A car covers 360 km in five hours. How much time would a bicycle take to cover the same distance traveling at one-fourth speed of the car?

(a) 18 hours

(b) 20 hours

(c) 22 hours

(d) 25 hours

Correct option is (b)

**Explanatory Answer:**

Speed of the car = 360/5 = 72 Kmph

Speed of the bicycle = 72 /4 = 18 Kmph.

So, time taken by motorbike = 360/18 = 20 hours

**Q4**. A Policeman observes a thief running at a speed of 60 kmph when the distance between them is 400m. The policeman started chasing him at the speed of 72 Km/h. In what time will the policeman catch the thief?

(a) 120 secs

(b) 130 secs

(c) 150 secs

(d) 60 secs

Correct option is (a)

**Explanatory Answer:**

The relative speed of the thief = (72-60) = 12 Kmph = 12 X (5/18) m/sec

Time taken by the police to catch the thief = 400 / {12 X (5/18)} = 120 Seconds

**Q5.** A, B and C started running around a circular stadium and completed one round in 24 seconds, 36 seconds and 30 seconds respectively. After what time will they meet again at the starting point?

(a) 12 min

(b) 6 min

(c) 18 min

(d) 24 min

Correct option is (b)

**Explanatory Answer:**

To meet again in the starting point, the

time taken will be LCM of (24,36 and 30

Seconds) which is 360 Seconds = 6 Minutes.

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| 2.5 Class Work Problems |

**Q1.** A man travels at a speed of 15m/s. How many kilometres will be cover in 4 hours 30 minutes.

(a) 250

(b) 243

(c) 265

(d) 283

**Q2**. Saroj reaches his office 2 hours late by travelling at 40 kmph from his home.

When he increases the speed to 60 kmph he is still late by 1 hour. What is the distance of his office from his home?

(a) 180

(b) 110

(c) 120

(d) 150

**Q3**. Shyam travelled from X to Y at 60km/h. Immediately after reaching Y, he travels towards X at a speed of 40 kmph. What is the average speed of the journey?

(a) 50 kmph

(b) 45 kmph

(c) 48 kmph

(d) Can’t say

**Q4.** Sara travels a certain distance at a speed of 30 kmph. She further travels a distance that is double the distance covered earlier at a speed of 40 kmph. What is the average speed of the journey?

(a) 36 kmph

(b) 32 kmph

(c) 34 kmph

(d) 38 kmph

**Q5**. Ram and Shyam start from a certain point at a speed of 30 kmph and 50 kmph in opposite directions. What will be the distance of separation between them after 3½ hours?

(a) 240 kms

(b) 260 kms

(c) 280 kms

(d) 300 kms

**Q6**. Bunty leaves from a certain place at 11.00 AM at 30 kmph. Babli starts from the same place at 3 PM at a speed of 60 kmph, chasing Bunty. When will Babli catch up with Bunty?

(a) 6 PM

(b) 7 PM

(c) 8 PM

(d) None of these

**Q7**. Had Bharath travelled 2km/hr faster to his office, he would have taken 6 minutes less. If Bharath had travelled 3 km/hr slower, he would have taken 12 minutes more. At what speed Bharath should travel so that he reaches his office just in time?

(a) 18 kmph

(b) 20 kmph

(c) 15 kmph

(d) Can’t say

**Q8**. Arul and Sajith start simultaneously and move towards each others from X and Y respectively. They met 40 kms from Y. The ratio of their speed is 1:5. Find the distance between them when they started?

(a) 45 kms

(b) 46 kms

(c) 47 kms

(d) None of these

**Q9**. Find the time (in seconds) that a train which is 350 m long travelling at 35 m/s take to cross a pole?

(a) 10 sec

(b) 12sec

(c) 15 sec

(d) 18 sec

**Q10.** In the above question # 9, find the time taken by the above train to cross a platform 175m long?

(a) 10 sec

(b) 12 sec

(c) 15 sec

(d) 18 sec

**Q11**. A train travelling at 36kmph crosses

a pole in 10 seconds. Find the time taken by this train to cross a platform which is 250m long?

(a) 25 sec

(b) 30 sec

(c) 35 sec

(d) 40 sec

**Q12.** Without stoppages, a train can cover 50 kms in an hour. With stoppages it can cover 30 kms in an hour. Find its stoppage time per hour in a journey it covers with stoppages (in minutes)

(a) 15

(b) 18

(c) 21

(d) 24

**Q13**. A train of length 200m running at a speed of 54 kmph crosses another train moving at a speed of 36kmph moving in a direction opposite to the former train in 20 seconds. What is the length of the second train?

(a) 500 m

(b) 250 m

(c) 300 m

(d) Can’t say

**Q14**. A train covers 90kms at a constant speed taking certain time. If the speed were increased by 5m/s, it will take 2.5 hours less to cover the same 90 kms. What is the speed at which the train is travelling now?

(a) 18 kmph

(b) 27 kmph

(c) 36 kmph

(d) 45 kmph

**Q15.** A train travels for 7 hours wherein the first half of the distance is covered at 30 kmph and the other half is covered at 40 kmph. Find the total distance travelled by the train?

(a) 125 kms

(b) 120kms

(c) 130 kms

(d) 115 kms

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| 2.6 Home Assignment |

**Q1**. How long will a train of length 250m, running at a speed of 50kmph take to cross a man running in the opposite direction at a speed of 22 kmph?

(a) 10 sec

(b) 12.5 sec

(c) 15 sec

(d) 17.5 sec.

**Q2.** The distance between Palghat and Cochin is 100 kms. A train started from Palghat towards Cochin at 9AM, at a speed of 20 kmph. After every 20 kms, the train halts for 15 mins at a station. At what time will this train reach Cochin?

(a) 3 pm

(b) 3.15 pm

(c) 3.30pm

(d) 2.45 pm

**Q3.** Joshi is on a pilgrimage to Sabarimala, which is located 80 kilometres away from his home. He wants to reach the destination in 7 hours. He plans to travel both on foot and on bus. He can walk at the rate of 8 km/hr. It being hilly terrain, the bus can travel only at 16 km/hr. What is the maximum distance that Joshy can travel on foot?

(a) 30 kms

(b) 46 kms

(c) 32 kms

(d) 20 kms

**Q4**. A train leaves a point X at 6 pm and reaches point B at 12 mid night. Another train leaves point B at 8 pm and reaches point A at 11 PM. At what time will these two trains meet?

(a) 9.20 pm

(b) 9.30 pm

(c) 9.15pm

(d) 10.05 pm

**Q5.** A man takes 5 hours in walking to a certain place and riding back. He would have gained 2 hours by riding both ways. The time he would take to walk both ways will be:

(a) 5.5 hrs

(b) 7 hrs

(c) 8 hrs

(d) 9.5 hrs

**Q6**. A train 300m in length crosses a pole in 30 seconds. Another train, travelling at the same speed that of the former train takes 50 seconds to cross a platform of length 250m. Find the time taken by the second train to cross the first train, assuming that the first train is stationery.

(a) 55 sec

(b) 27.5 sec

(c) 50 sec

(d) 25 sec

**Q7.** Walking at of his usual speed, a man is 20 minutes too late. What is the usual time taken by him to cover that distance?

(a) 1 Hr

(b) 1.5 hrs

(c) 2 hrs

(d) 3.5 hrs

**Q8.** In a 100m race, A beats B by 5m and C

by 15m. By how many metres would B beat C in a race of 2850 metres?

(a) 350m

(b) 300m

(c) 285 m

(d) 325m

**Q9.** In a 600m race, Ram beats Shyam by 150 metres or 30 seconds. Find the speed at which Shyam runs?

(a) 5 m/s

(b) 8 m/s

(c) 12 m/s

(d) 10 m/s

**Q10**. A man rows downstream and covers 15 kms in 3 hours. He covers the same distance upstream in 5 hours. What is the difference in the distance travelled upstream in 10 hours and that downstream in 10 hours?

(a) 18 kms

(b) 20 kms

(c) 25 kms

(d) 28 kms

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| 2.7 Self Assessment |

**Q1.** In a 100 metres race, A moves at 8 Kmph. If A gives B a start of 6 metres and still beats B by 2 seconds. What is the speed at which B is moving?

(a) 2 m/s

(b) 2.5 m/s

(c) 3 m/s

(d) None of these

**Q2.** A, B and C are three contestants in a 2kilometer race. If A can give B a start of 80 m and A can give C a start of 128 m,

how many metres start can B give C?

(a) 50m

(b) 75m

(c) 80m

(d) None of these

**Q3.** In a 1000m race, the ratio of the speeds of two contestants A and B is 3:4. A has a start of 280 metres. Then A wins by \_\_\_ metres.

(a) 60

(b) 45

(c) 42

(d) 40

**Q4.** A boat can cover 40 Kms downstream in 4 hours and 6 Kms upstream in 3 hours. Find the time it would take to cover a round-trip journey between two points 20 Kms apart?

(a) 14 Hours

(b) 12 Hours

(c) 15 Hours

(d) 10 Hours

**Q5.** A stream is flowing at 12 Kmph. Speed of boat is 25% more than that of the stream. Find the average speed (in kmph) of the boat in completing a round trip journey between two points 81 Kms apart?

(a) 5

(b) 5

(c) 5

(d) 5

**Q6.** Sachin and Sourav were practicing on a circular track of length 2000 metres. Sachin can run at a speed of 24 Kmph and Sourav can run at a speed of 27 Kmph, around the track. Find, when will they meet for the first time at the starting point?

(a) 30 Mins

(b) 35 Mins

(c) 40 Mins

(d) 45 Mins

**Directions for Questions( Q7& Q8):**

Sachin and Sourav were practicing on a circular track of length 2000 metres. Sachin can run at a speed of 10 m/s and Sourav can run at a speed of 15 m/s around the track.

**Q7**. If they were running in opposite

directions, when will they meet for the first time in the track?

(a) 1 min 20 Sec

(b) 1 min 30 Sec

(c) 6 min 40 Sec

(d) None of these

**Q8**. If they were running in same

directions, when will they meet for the first time in the track?

(a) 1 min 20 Sec

(b) 1 min 30 Sec

(c) 6 min 20 Sec

(d) 6 min 40 Sec

**Q9.** The speed of two persons are in the

ratio 6:5. They started travelling in opposite directions from the same point. After 4 hours, they were 308 Kilometres apart. Find the speed of the slower person in kmph.

(a) 30

(b) 35

(c) 40

(d) 42

**Q10.** Usain Bolt ran against the wind, then turned back and ran for half the time he earlier ran. The distance he covered with the wind was thrice the distance he covered against the wind. If the speed of the wind was 6 Kmph, what is the usual speed of Usain in Kmph?

(a) 9

(b) 8.4

(c) 7.2

(d) 6.3

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| 2.8 Additional Exercise from various competitive examinations |

**Q1**. Laxman and Bharat decide to go from Agra to Delhi for watching a cricket match and board two different trains for that purpose, While Laxman takes the first train that leaves for Delhi, Bharat decides to wait for some time and take a faster train. On the way, Laxman sitting by the window seat noticed that the train boarded by Bharat crossed him in 12 seconds. Now the faster train can travel 180 km in three hours, while the slower train takes twice as much time to do it. Given this, mark all the correct options.

(a) If thefaster train has taken 30 seconds to cross the entire length of the slower train, the difference between the lengths of the two trains is 50 m.

(b) If the faster train had been running

twice as much faster, it would have taken 10 seconds to overtake the slower train.

(c) Had the faster train taken 24 seconds

to cross the entire length of the slower train, the length of the slower train would have been 100 m.

(d) If the slower train had been running at one and a half times of its current speed, the faster train would have taken 24 seconds to overtake Laxman (IIFT)

**Q2.** Ranjan goes to a countryside lake for a boat ride. Standing at the ferry counter, he looked at the opposite bank and observed a tall tower on a hill downstream, the angle of elevation being 45°. Ranjan comes to know from the bystanders that the tower is a historical ruin and decides to visit it. The boat takes him directly to the opposite bank, from where the angle of elevation to the top of the tower becomes 60°. While exploring the site, he comes to know that the combined height of the tower and the hill is 300 m. It the speed of the boat by which Ranjan travelled was 2 km/hr in still waters, mark the *correct* observations,

(a) It took Ranjan 3√6 minutes to cross the lake by the boat.

(b) The breadth of the lake is 100√6 metres

(c) It took Ranjan 4√3 minutes to cross the lake by the boat.

(d) If the combined height of the hill and the tower was 450 m and the speed of the boat was 1 km/hr (the angles of elevation remaining unchanged) , the time taken by Ranjan to cross the lake by boat would have been 9 √6 minutes. (IIFT)

**Q3**. A cyclist drove one kilometer, with the wind in his back, in three minutes and drove the same way back, against the wind in four minutes. If we assume that the cyclist always puts constant force on the pedals, how much time would it take him to drive one kilometer without wind?

(a) 2

(b) 3

(c) 2

(d) 3 (SNAP)

**Q4.** A flight of Jet Airways from Delhi to Mumbai has an average speed of 700 kilometres per hour without any stoppage, whereas a flight Kingfisher from Delhi to Mumbai has an average speed of 560 kilometres per hour with stoppage at Baroda. What is the average stoppage time per hour of Kingfisher flight if both the planes fly at the same speed?

(a) 8 minutes

(b) 12 minutes

(c) 16 minutes

(d) 24 minutes (FMS )

**Q5.** A 500-gram stone was dropped from the roof of a building. What is the height of the building if the stone reached the ground in 4 seconds?

(a) 108,4 metres

(b) 98.4 metres

(c) 88.4 metres

(d) 78.4 metres (FMS)

**Directions for Questions ( Q6- Q9):**

Everyday Miss Yadav, Miss Sharma, Miss Toppo and Miss Hussain go to a park for morning walk. One day, they reach the gate of the park at the same time and in immediately start walking on the only circular track adjacent to the gate. Miss Yadav, Miss Toppo and Miss Hussain go on a clockwise direction while Miss Sharma goes anticlockwise. Miss Hussain who is asthmatic is the slowest among the four and soon others move away from her. Like every day she could walk only one round taking almost the same time as others to complete the morning walk. After her walk Miss Hussain read the following instruction written at the gate while others join her one after another. "Walkers are requested to use only the 500 m walking track, Plucking of flowers and leaves are strictly prohibited. Park will remain closed from 6 pm to 5 am.”

While walking Miss Yadav overtakes Miss Hussain twice- once near the fountain and other time at the signature rock. Miss Toppo and Miss Sharma cross her three times (FMS)

**Q6.** What is the total distance covered by Miss Sharma and Miss Toppo together ?

(a) 3500 m

(b) 4000m

(c) 2500m

(d) 3000 m

**Q7**. How many times Miss Yadav and Miss Sharma cross each other on the track?

(a) Twice

(b) Three times

(c) Four times

(d) Six times

**Q8.** How many times Miss Toppo would overtake Miss Yadav?

(a) Never

(b) Once

(c) Twice

(d) Three times

**Q9.** The speed of a railway engine is 42 Km per hour when no compartment is attached, and the reduction in speed is directly proportional to the square root of the number of compartments attached. If the speed of the train carried by this engine is 24 Km per hour with 9 compartments are attached, the maximum number of compartments carried by the engine is:

(a) 49

(b) 48

(c) 46

(d) 47 (CAT)

**Q10.** Navjivan express from Ahmedabad to Chennai leaves Ahmedabad at 6.30 am and travels at 50km per hour towards Baroda situated 100 kms away. At 7:00 am Howrah - Ahmedabad express leaves Baroda towards Ahmedabad and travels at 40 km per hour. At 7.30 am. Mr. Shah. the traffic controller at Baroda realises that both the trains are running on a same track How much time does he have to avert a head-on collision between the two trains?,

(a) 15 min

(b) 20 min

(c) 25 min

(d) 30 min (CAT)

**Q11.** Rahim plans to drive from city A to station C, at the speed of 70 km per hour, to catch a train arriving there from B. He must reach C at least 15 minutes before the arrival of the train. The train leaves B, located 500 km south of A, at 8.00 am and travels at a speed of 50 km per hour. It is known that C is located between west and northwest of B, with BC at 60° to AB. Also, C is located between south and southwest of A with AC at 30° to AB. The latest time by which Rahim must leave A and still catch the train is closest to

(a) 6.15 am

(b) 6:30 am

(c) 6:45 am

(d) 7:00 am

(e) 7:15 am (CAT 2008)

**Directions for Questions (Q12 & Q13):**

Cities A and B are in different time zones. A is located 3000 km east of B. The table below describes the schedule of an airline operating non-stop flights between A and B. All the times indicated are local and on the same day.

|  |  |  |  |
| --- | --- | --- | --- |
| Departure | | Arrival | |
| City | Time | City | Time |
| B | 8.00am | A | 3.00pm |
| A | 4.00pm | B | 8.00pm |

Assume that planes cruise at the same speed in both directions. However, the effective speed is influenced by a steady wind blowing from east to west at 50 km per hour. (CAT)

**Q12.** What is the time difference between A and B?

(a) 1 hour and 30 minutes

(b) 2 hours

(c) 2 hours and 30 minutes

(d) 1 hour

(e) Cannot be determined

**Q13.** What is the plane's cruising speed in km per hour?

(a) 700

(b) 550

(c) 600

(d) 500

(e) Can’t be determined

**Q14.** Arun, Barun and Kiranmala start from the same place and travel in the same direction at speeds of 30, 40 and 60 km per hour respectively. Barun starts two hours after Arun. If Barun and Kiranmala overtake Arun at the same instant, how many hours after Arun did Kiramala start'?

(a) 3

(b) 3.5

(c) 4

(d) 4.5

(e) 5 (CAT)

**Q15.** A jogging park has two identical circular tracks touching each other, and a rectangular track enclosing the two circles. The edges of the rectangles are tangential to the circles. Two friends, A and B, start jogging simultaneously from the point where one of the circular tracks touches the smaller side of the rectangular track. A jogs along the rectangular track, while B jogs along the two circular tracks in a figure of eight. Approximately, how much faster than A does B have to run, so that they take the same time to return to their starting point?

(a) 3.68%

(b) 4.22%

(c) 4.44%

(d) 4.72% (CAT)

**Chapter 3**

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| MEN, WORK AND TIME |

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| 3.1 Learning objective: |

In this chapter, problems related to men, work and time, efficiency, performance will be discussed.

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| 3.2 Introduction: |

Performing or doing work of any amount involves efforts of person(s) over a period of time. Therefore, the number of persons (P) , the quantity of work (W) and the period of time taken (T) are important variables in problems related to Time and Work. Moreover time (T) taken to do a work depends not only on how many persons are employed to do it but also on how efficient they are.

**Efficiency**

Efficiency here means *rate of doing same work.* This aspect comes into picture when the problem involves comparison of works done by different categories of persons. For instance, efficiencies of man, woman, boy, girl in general are different. Even efficiency of one man may not be same as that of other; but unless otherwise specifically stated in the problem, all men or women working in a group are assumed to do work with equal efficiency.

**Methods for solving problems**

For solving questions based on time and

work, following two methods are widely used.

1. **Ratio and Proportion Method**:

Since the problem concerning to Time and Work have proportional relation, these can be solved by this method also.

**Unitary Method:**

This is a very simple and useful method. The term 'unitary' is self-evident. In this method we first proceed to reduce the problem to either work done by one person or work done in 1 day and so on as per the requirement of the problem.

**Illustration**

Assume that *A* can complete a work in 2 days. So, on day 1, *A* will do half of the work and on day 2 he will do the remaining half of the work. Now, suppose *B* too alone can do a piece of work in 2 days. So *B* will do half of the work on 1st day and half of the work on 2nd day. Now, let *A* and *B* start together. On day 1 *A* will do half of the work and *B* also does half of the work.

So, half work +half work = one work is completed by *A* and *B* on the day 1 itself.

Following simple facts must be kept in mind while solving problems on Time and Work.

1. The efficiency of workmen remains same while working. In other words, each workman does the same work each day.

2. The total work is assumed to be 1.

3. If a man can do a piece of work in n days, then on a single day he completes of the total work. Conversely if a man can do of a work on a single day, he works for p days for completing the work.

4. If the number of men engaged to do a piece of work be changed in the ratio *m: n* the time required for. the work will be changed in the inverse ratio *n : m.*

5. If *A* is *x* times as good a work man as B, then *A* will take of the time taken by B for doing the work. In other words, if

A's work : Bs work = 2 : 1 , then ratio of the time taken by A and B to finish the work = 1 : 2

i.e. then A will take ½ of the time taken by *B* to finish that work.

6. From the above facts we may derive that work, workman and time bear the following relations

(a) More men, less days and conversely more days, less men.

(Inverse proportion)

(b) More men, more work and conversely more work, more men.

(Direct proportion)

(c) More days, more work and conversely more work, more days .

(Direct proportion)

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| 3.3 Important formulae |

1. If A can finish a piece of work in n days, then A's 1 day's work is (1/n).

2. If M1 persons can do `W1' works in D1

days for T1 hours and M2 persons can do

`W2' works in D2 days for T2 hours then

3. If A can finish a work in x days and B can finish the same work in y days, then time taken by both to finish the work is days

4. If A, B and C can do a work in x, y, and z days respectively, then all of them working together can finish the work in days

|  |
| --- |
| 3.4 Quiz |

**Q1.** A can do a piece of work in 8 days, while B and C can do the same work in 20 and 40 days respectively. In how many days can all the three of them together complete a work which is four times that of the previous work?

(a) 25

(b) 24

(c) 20

(d) 15

**Q2**. In 4 hours Satish can type 60 pages while Shyam can type only 40 pages. Find in how many hours will they type 250 pages?

(a) 25

(b) 12

(c) 8

(d) None of these

**Q3.** For constructing a bridge, Ramprasad has employed 25 men so that they would finish a piece of work in 120 days. At the end of every 10 days, 10 additional men are employed, in how many days will it be

finished?

(a) 50

(b) 55

(c) 60

(d) 75

**Q4.** An empty tank can be filled by pipe A in 4 hours and by pipe B in 6 hours. If the two pipes are opened for 1 hour each alternately with A as the first opening

pipe then in how many hours will the tank be filled

(a) 1

(b) 2

(c) 4

(d) 5

**Q5**. Three pipes-A, B and C can fill a tank from empty to fill in 30mins, 20mins and 10 mins respectively. When the tank is empty all the three pipes are opened and

A ,B and C discharge chemicals P,Q and R respectively. After 3 minutes, what is the proportion of solution R in the liquid?

(a)

(b)

(c)

(d)

|  |
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| 3.5 Worked Out Examples |

**Q1.** 24 men can complete a work in 80 days. How many men can complete the same work in 30 days?

(a) 50

(b) 60

(c) 64

(d) 80

Correct option is (c)

**Explanatory Answer:**

Let ‘m’ be the number of men required.

We have m X 30 = 24 X 80. Solving, we have m = 64

**Q2.** 25 men can paint a wall in 24 hours and 36 women can paint the same wall in 30 hours. In how many hours can 5 men and 6 women paint the wall together?

(a) 54

(b) 60

(c) 72

(d) 90

Correct option is (c)

**Explanatory Answer:**

25 x 24 Man hours = 36 x 30 woman hours.

5 Men = 9 Women

5 Men + 6 Women = 9 Women + 6 Women = 15 Women

15 x H = 36 X 30

So, H = 72 Hours

**Q3**. Pipe A and pipe B can fill a tank in 20 minutes and 40 minutes respectively. Another pipe C can empty the full tank in 80 minutes. Find the time taken to fill the empty tank when all the three pipes are opened simultaneously.

(a) 10 min

(b) 12 min

(c) 16 min

(d) 20 min

Correct option is (c)

**Explanatory Answer:**

When all the three pipes are opened, (1/20) + (1/40) -(1/80) of the tank is filled in a minute => (1/16) th of the tank in a minute. Thus, the tank will be filled in 16 minutes.

**Q4.** A single person takes 7 minutes to type a page. If from 1 pm to 2 : 45 pm 3375 pages are to be typed, how many persons should be employed on this job?

(a) 215

(b) 225

(c) 240

(d) 255

Correct option is (b)

**Explanatory Answer:**

We have M1 = 1, H1 = 7 Mins and W1 = 1

M2 = ?, H2= 105 Minutes and W2 = 3375

Solving, we have M2 = 225

**Q5.** Two men and two women together can complete a job in 5 days. A woman can complete the job in 30 days. Find the time taken by a man to complete the job.

(a) 20 days

(b) 15 days

(c) 12 days

(d) 18 days

Correct option is (b)

**Explanatory Answer:**

Two men and two women together can complete a job in 5 days

One man and one woman together can complete a job in 10 days

They do of the job in a day

A woman can do of job in a day. So a man can do- of job in a day

A man can do of a job in a day

A man can do the job in 15 days.

|  |
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| 3.6 Class Work |

**Q1.** Ajay, Vijay and Sujay can complete a piece of work by working together in 6 days. If Vijay and Sujay can complete the work in 15 days, while Ajay and Vijay can complete the work in days, in how many days can Ajay complete the work working independently?

(a) 10

(b) 20

(c) 60

(d) None of these

**Q2.** A team of 1000 men can construct a

wall of length 2000m in 40 days. In how many days can of the present team construct a wall of length 2500m working more time per day?

(a) 150

(b) 125

(c) 100

(d) 95

**Q3.** 45 men can make 140 pots in 12 days, working 10 hours per day. 15 men leaves the work after working for 6 days. How many hours per day should the rest of the team work to complete the work as per schedule?

(a) 12

(b) 13

(c) 14

(d) 15

**Q4.** Ajay, Babu and Chithra can complete a piece of work in 8, 12 and 24 days respectively. Babu joined Ajay and Chithra after one day of commencement of the work. Chithra fell ill and she left the team 2 days before the completion of the work. In how many days did the work get completed?

(a) 3

(b) 3

(c) 4

(d) 4

**Q5.** Two pipes P and Q can fill a tank in 20mins and 30 mins respectively. If both the pipes are used together how long (in mins) will it take to fill a tank which is thrice the capacity of the former tank?

(a) 12

(b) 24

(c) 36

(d) None of these

**Q6.** A tank can be filled by a tap in 9 hours while it can be emptied by another tap in 4 hours. Both the taps are opened simultaneously. After how much time will the tank get filled?

(a) 7.2 hrs

(b) 6.5 hrs

(c) 5 hrs   
(d) None of these

**Q7.** A water tank is 3/5th empty. Pipe A can fill a tank in 10 minutes and pipe B can empty it in 6 minutes. Both the pipes are opened. How long will it take to empty/fill the tank completely?

(a) 6 mins to fill

(b) 9 mins to fill

(c) 6 mins to empty

(d) 9 mins to empty

**Q8.** A large swimming pool can be filled by two pipes A and B in 60 minutes and 40minutes respectively. How many minutes will it take to fill the pool from empty state if B is used for half the time and A and B fill it together for the other half?

(a) 15

(b) 20

(c) 25

(d) 30

**Q9.** Three taps- P,Q and R can fill a tank in 12,15 and 20 hours respectively. Assume P is kept open throughout Q and R are open for one hour each alternate the tank will be fill in \_\_\_\_?

(a) 5Hrs

(b) 6Hrs

(c) 7Hrs

(d) None of these

**Q10.** 6 men or 12 women or 18 boys can complete a job in 10 days. How many women should accompany 2 men and 3 boys so that the work gets completed in 10 days?

(a) 10

(b) 6

(c) 12

(d) 4

**Q11.** If 10 men or 16 women can do a piece of work in 21 days, then in how many days can 15 men and 16 women do the same work?

(a) 6

(b) 9

(c) 8

(d) 10

**Q12.** Two men and six boys can complete a task in 5 days. One woman alone can complete the same work in 20 days. How many women should accompany 3 men and 9 boys so that the work gets completed in 2 days?

(a) 4

(b) 2

(c) 3

(d) 6

**Q13.** Wages for 30 women is Rs. 80,000 for 36 days. How many women must accompany 10 men to complete a piece of work in 18 days, if it is known that men earn double that of a women?

(a) 30

(b) 40

(c) 50

(d) 60

**Q14.** Aravind, Bala and Chandu can do a piece of work in 24, 30 and 40 days respectively. They start to work together but Chandu leaves 4 days before the completion of the work. In how many days is the work done?

(a) 8 days

(b) 9 days

(c) 10 days

(d) 11 days

**Q15.** A tap P can fill a tank in 20 hours. Another tap Q can fill the tank in 30hours. Both the tanks are opened at 3.00 AM and after sometime Q is closed. If the tank is filled by 6.00pm, then at what time Q was closed?

(a) 4.30pm

(b) 6.30pm

(c) 8.30pm

(d) 10.30am

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| 3.7 Home Assignment |

**Q1**. A group of men takes certain time to complete a task. If there were 30 men more, they would have taken 8 days less to complete the task. If there were 20 men less, they would have taken 8 more days to complete the same task. Find the number of men in that group?

(a) 100

(b) 110

(c) 120

(d) 140

**Q2**. 8 men working for 10 hours a day can dig a well in 60 days. How many men need to be employed to dig the well in 25 days if the working hours are revised as 8 hours per day?

(a) 30

(b) 40

(c) 24

(d) 18

**Q3.** Twenty men can complete a job in 100 days. They started the work and continued to work for 20 days. After that, 10 men with four times the normal efficiency joined them. In how many more days will the work get completed?

(a) 16

(b) 20

(c) 30

(d) 40

**Q4.** Raja and Rani can complete a job in α times the time that Mantri would take. Raja and Mantri can complete a job in α times the time that Rani would take. Rani and Mantri can complete a job in α times the time that Raja would take. What is the value for α?

(a) 4

(b) 2

(c)

(d)

**Q5.** Santa can complete a job in 12 days. Banta can complete the job in 8 days. They worked together for 4 days. The remaining part of the work was completed by Sabha in 3 days. The total wage paid to Santa, Banta and Sabha was Rs.10,800. What was the share of Sabha?

(a) 1500

(b) 1600

(c) 1800

(d) 2400

**Q6.** Ram and Shyam together can do a piece of work in 7 days. If Ram alone does twice as much work as Shyam in a given time, find how long Ram alone would take to do the work?

(a) 9.5 days

(b) 10.5days

(c) 11.5days

(d) None of these

**Q7.** Soujanya and Sagarika can do a piece of work in 12 days, Sagarika and Sushama in 15 days, Soujanya and Sushama in 20 days, How long would all take to finish the

work together?

(a) 15days

(b) 14 days

(c) 10 days

(d) 8 days

**Q8**. Anand can do as much work in 2 days as Bhagya in 3 days and Bhagya as much in 4 days as Chameli in 5 days. In what time could Anand, Bhagya and Chameli together do a piece of work when Anand can do it in 11 days?

(a) 6 days

(b) 7 days

(c) 5 days

(d) Can’t Say

**Q9.** Pipe A can be used for filling as well as for emptying at the same capacity. Pipe B is a filling pipe and it would take ‘Z’ hours to fill a tank. Assume that pipe A is used as an emptying pipe along with B, it would take ‘3Z’ hours to fill the tank. Find the ratio of the times that A and B would take to fill the tank?

(a) 2:3

(b) 3:2

(c) 3:1

(d) 1:3

**Q10.** Two women and three boys can do a piece of work in 8 days and three women and two boys can do it in 7 days. If the daily wages of a boy be Rs.18. what will be the weekly wages of a woman?

(a) Rs. 250

(b) Rs36

(c) Rs.252

(d) None of these

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| 3.8 Self assessment |

**Q1.** Anand can do a piece of work in 36 days, Bhaskar in 54 days and Govind in 72 days. All the three began the work together but Anand got fed up and left after 8 days of work. B got a better option to work so he also left 12days before the completion of the work. How many days in all did Govind put in till the entire work was finished?

(a) 20 days

(b) 35 days

(c) 40days

(d) None of these

**Q2.** If two women or three children can do a work in 35 days, in how many days 1 woman and 9 children can do that work?

(a) 8 days

(b) 10 days

(c) 12 days

(d) 15 days

**Q3.** Peter can do a piece of work in 90 days, Bose in 40 days and Gopi in 12days. They work for a day each in turn i.e. first day Peter does it alone, Bose does it the second day and Gopi the third day. After that Peter does it for another day and the cycle continues. After finishing the work they got Rs.240 as wages. If the wages are divided in proportional to the work done by them, find the share of peter?

(a) Rs.30

(b) Rs.45

(c) Rs.25

(d) Rs.24

**Q4.** Rathna takes twice as long as Radha and Roopa together take to do a piece of work and Radha takes twice as long as Rathna and Roopa together. However they together can complete the work in 30days. How long would Roopa alone take to do the work?

(a) 100 days

(b) 90 days

(c) 85 days

(d) 70 days

**Q5.** A and B can do a work in 10days. B and C can do the same work in 20days.while C and A can do it in 15 days. In how many days can C alone do the same work?.

(a) 100

(b) 120

(c) 130

(d) 140

**Q6.** A can do a piece of work in 10 days and B in 20days. They work together but 2 days before the completion of the work. A leaves in how many days was the work completed?

(a) 6days

(b) 8days

(c) 10days

(d) None of these

**Q7.** A and B could do a piece of work in 30days. After working for 10days, they are assisted by C and the work is finished in 10days. If C does as much work in 2days as B does in 3days. In how many days could A do the same work alone?

(a) 80 days

(b) 90 days

(c) 120 days

(d) 45 days

**Q8.** If 8 men and 12 boys can complete a piece of work in 12 days, in what time will 40 men and 45boys complete another piece of work three times as great, supposing 16 men can do as much work in 8 hours as 12 boys can do in 24 hours?.

(a) 8 days

(b) 10 days

(c) 12 days

(d) 15 days

**Q9.** A is twice as good a workman as B and together they can complete a piece of work in 14 days. In how many days can it be done by A alone?

(a) 18

(b) 20

(c) 25

(d) 21

**Q10.** Working 5 hours a day, A can

complete a work in 8 days and working 6 hours a day, B can complete the same work in 10 days. working 8 hours a day they can jointly complete the work in

(a) 3 days

(b) 4 days

(c) 4.5 days

(d) 5.4 days

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| 3.9 Additional Exercise questions  from various competitive exams |

**Q1.** A contractor takes up an assignment that 20 men can complete in 10 days. The same assignment could be finished by 15 women in 20 days. The contractor decides to employ 10 men and 10 women for the project. Given this, mark all the correct options.

(a) If the wage rate for men and women are Rs 50 and Rs.45 respectively the total wage bill for the product will be Rs.11,400.

(b) If the wage rate for men and women are Rs.45 and Rs.40 respectively the total wage bill for the project will be Rs 10,200.

(c) If the wage rate for men and women are equal at Rs. 40, the total wage bill for the project will be Rs. 9,100.

(d) If the contractor decides to employ 20 men and 30 women for the project and the wage rate for men and women are Rs. 40 and Rs. 35 respectively, the total wage bill for the project will be Rs. 9,250. (IIFT )

**Q2.** Pavan builds an overhead tank in his house, which has three taps attached to it. While the first tap can fill the tank in 12 hours, the second one takes one and a half times more than the first one to fill it completely. A third tap is attached to the tank which empties it in 36 hours. Now, one day in order to fill the tank. Pavan opens the first lap and after two hours opens the second tap as well. However, at the end of the sixth hour, he realizes that the third tap has been kept open right from the beginning and promptly closes it. What will be the total time required to fill the tank?

(a) 8 hours 48 minutes

(b) 9 hours 12 minutes

(c) 9 hours 36 minutes

(d) 8 hours 30 minutes (IIFT)

**Q3**. A contract is to be completed in 50 days and 105 men were set to work, each working 8 hours a day. After 25 days. 2/5th of the work is finished. How many additional men be employed so that the work may be completed on time, each man now working 9 hours a day?

(a) 34

(b) 36

(c) 35

(d) 37 (SNAP)

**Q4.** A can build up a structure in 8 days and B can break it in 3 days. A has worked for 4 days and then B joined to work with A for another 2 days only. In how many days will A alone build up the remaining part of the structure?

(a) 10 days

(b) 9 days

(c) 12 days

(d) None of these (SNAP)

**Q5.** A worker working under a bonus scheme saves 10 hours in a job for which the standard time is 60 hours. A bonus of 10% of the hourly rate is payable when he reaches 100% efficiency. An additional bonus of 2% of the hourly rate for each 1% in excess of 100% efficiency is given. If the normal wage rate is Rs.2 per hour, find the wages payable to the worker

(a) Rs 120

(b) Rs.130

(c) Rs 150

(d) None of these (JMET)

**Q6**. Delhi Metro Corporation engaged 25.000 workers to complete the project of lP state to Dwarka Metro Line in 4 years. At the end of the first year 10% workers were shifted to the other projects of Delhi Metro. At the end of second year again 5% workers were reduced. However, the number of workers increased by 10% at the end of the third year to complete the above project in time. What was the strength of work force during the fourth year?

(a) 23145

(b) 23131

(c) 23512

(d) 23513 (FMS)

**Q7**. Due to the recent global financial crisis, many companies in the field of IT Services offer jobs on contractual basis with a clause of fine for the leave taken by the employee. An ITS company employs an engineer for 290 days on a salary of Rs 500 for eight hours work per day, and the engineer has to pay a fine of R 50 for each hour of his absence. The engineer may compensate his one-day absence by working 4 hours extra for two days. If the engineer receives Rs.132400 at the end of the contract how many hours was he absent from his job?

(a) 110 hours

(b) 112 hours

(c) 114 hours

(d) I18 hours (FMS)

**Q8.** A telecom service provider engages male and female operators for answering 1000 calls per day. A male operator can handle 40 calls per day whereas a female operator can handle 50 calls per day. The male and female operators get a fixed wage of Rs 250 and Rs 300 per day respectively. In addition, a male operator gets Rs.15 per call he answers and a female operator gets Rs.10 per call she answers. To minimize the total cost how many male operators should the service provider employ assuming he has to employ more than 7 of the 12 female operators available for the job?

(a) 15

(b) 14

(c) 12

(d) 10 (CAT )

**Q9.** In Nuts and Bolts factory, one machine produces only nuts at the rate of 100 nuts per minute and needs to be cleaned for 5 minutes after production of every 1000 nuts. Another machine produces only bolts at the rate of 75 bolts per minute and needs to be cleaned for 10 minutes after production of every 1500 bolts. If both the machines start production at the same time, what is the minimum duration required for producing 9000 pairs of nuts and bolts?

(a) 130 minutes

(b) 135 minutes

(c) 170 minutes

(d) 180 minutes (CAT)

**Q10.** A leather factory produces two kinds of bags- standard and deluxe. The profit margin is Rs20 on a standard bag and Rs.30 on a deluxe bag Every bag must be processed on machine A and on machine B. The processing time per Bag on the two machines are as follows.

|  |  |  |
| --- | --- | --- |
|  | Time required (Hour per bag) | |
|  | Machine A | Machine B |
| Standard Bag | 4 | 6 |
| Deluxe Bag | 5 | 10 |

The total time available on machine A is

700 hours and on machine B is 1250

hours. Among the following production plans. which one meets the machines availability constraints and maximizes the profit?

(a) Standard 75 bag & Deluxe 80 bag

(b) Standard 100 bags, Deluxe 60 bags

(c) Standard 50 bags Deluxe 100 bags

(d) Standard 60 bags. Deluxe 90 bags (CAT)

**Q11.** 3 small pumps and a large pump are filling a tank. Each of the three small pumps works at 2/3rd the rate of the large pumps. If all 4 pumps work at the same time, they should fill the tank in what fraction of the time that it would have taken the large pump alone?

(a)

(b)

(c)

(d) (CAT)

**Q12.** A can complete a piece of work in 4 days. B takes double the time taken by A. C takes double that of B and D takes double that of C to complete the same task. They are paired in groups of two each. One pair takes two-thirds the time needed by the second pair to complete the work. Which is the first pair?

(a) A, B

(b) A, C

(c) B, C

(d) A, D (CAT)

**Q13.** There’s a lot of work in preparing a birthday dinner. Even after the turkey is in the oven, there's still the potatoes and gravy, yams, salad and cranberries not to mention setting the table. Three friends Asit, Arnold and Afzal work together to get all of these chores done. The time it takes them to do the work together is 6 hrs less than Asit would have taken alone, one hour less than Arnold would have taken working alone and half the time Afzal would have taken working alone. How long did it take them to do these chores working together?

(a) 20 minutes

(b) 30 minutes

(c) 40 minutes

(d) 50 minutes (CAT)

**Q14.** Ramesh has two examinations on Wednesday- Engineering Mathematics in the morning and Engineering Drawing the afternoon. He has a fixed amount of time to read the text books of both these subjects on Tuesday. During this time he can read 80 pages of Engineering Mathematics and 100 pages of Engineering Drawing. Alternatively he can also read 50 pages of Engineering Mathematics and 250 pages of Engineering drawing. Assume that the amount of time it takes to read one page of the text book of either subject is constant. Ramesh is confident about Engineering Drawing and wants to devote full time to reading Engineering Mathematics. The number of Engineering Mathematics text book pages he can read on Tuesday is

(a) 500

(b) 300

(c) 100

(d) 60 (JMET)

**Q15.** Rohit, Harsha and Sanjeev are three typists who, working simultaneously, can type 216 pages in four hours. In one hour, Sanjeev can type as many pages more than Harsha as Harsha can type more than Rohit. During a period of five hours, Sanjeev can type as many pages as Rohit can during seven hours. How many page does each of them type per hour?

(a) 16, 18, 22

(b) 14, 17, 20

(c) 15, 17, 22

(d) 15, 18, 21 (MAT)

**Chapter 4**

|  |
| --- |
| Heights and Distances |

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| 4.1 Learning objective: |

By the end of the lesson you will be able to understand the terminologies used in heights and distances and be able to solve questions on heights and distances.

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| --- |
| 4.2 Introduction: |

|  |  |
| --- | --- |
| Angle of Elevation*:* | Angle of Depression: |
| http://www.indiabix.com/_files/images/aptitude/1-zf-34-02.gif | http://www.indiabix.com/_files/images/aptitude/1-zf-34-03.gif |
| Suppose a man from a point O looks up at an object P, placed above the level of his eye. Then, the angle which the line of sight makes with the horizontal through O, is called the *angle of elevation* of P as seen from O.  http://www.indiabix.com/_files/images/aptitude/1-sym-tfr.gifAngle of elevation of P from O = http://www.indiabix.com/_files/images/aptitude/1-sym-ang.gifAOP. | Suppose a man from a point O looks down at an object P, placed below the level of his eye, then the angle which the line of sight makes with the horizontal through O, is called the *angle of depression* of P as seen from O.  http://www.indiabix.com/_files/images/aptitude/1-sym-tfr.gifAngle of depression of P from O = http://www.indiabix.com/_files/images/aptitude/1-sym-ang.gifAOP. |

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| 4.3 Procedure: |

Step 1: Read the question and draw the diagram.

Step 2: Get the data and fill it on the diagram.

Step 3: Find the required whit the help of trigonometric ratios.

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| 4.4 Worked out examples |

**Q1.** The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Correct answer is

**Explanatory Answer:**

300

600

A

C

B

D

50m

In △ABD, we have

tan 600=  **(1)**

In △BCD, we have

Tan300= (2)

On solving we get CD =

CD =

**Q2.** A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.

**Explanatory Answer:**

300

600

A

D

C

B

20m

It is given that **BC = 20 m**

In △*ADC*,

Tan300 = (1)

In △*ABD*

Tan600 = (2)

Dividing these two equations we get,

Solving we get, CD =10m.

**Q3.** A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

**Explanatory Answer:**

It is given that car travels from point D to point C in 6 seconds.

In △*ABC*, we have

tan600   =**(1)**

In △*ABD*, we have

Tan 300=        **(2)**

C

600

300

300

600

B

D

A

From **(1) and (2),** we get

=3

⇒=2

**Therefore, length of CD is twice as length of BC.**

Car is traveling at uniform speed and it is given that car takes 6 seconds to move from point D to C. **Therefore, the time required to move from point C to B would be 3 seconds.**

**Q4.** From the top of a 7 m building, the angle of elevation of the top of a cable tower is 60∘ and the angle of depression of its foot is 45∘. Determine the height of the tower.

**Explanatory Answer:**

It is given that height of building **(BC)**

= 7 m

We want to find height of tower **AD**.

In △*BCD*

=tan450

⇒CD=7 m

We have **CD = BE,**

Therefore, **BE=7** m

600

450

7m

A

E

D

C

B

7m

In △*ABE*

**=**tan60∘

⇒AE=BE×tan60∘

⇒AE=7×√3=7√3 m

**Height of Tower (AD)**

**= AE + DE =** 7√3+7=7(√3+1) m

**Q5.** The angles of elevation of the top of

a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complimentary. Prove that the height of the tower is 6 m.

**Explanatory Answer:**

A

C

D

B

X0

(90-X)0

4m

5m

It is given that ∠ADB and ∠ACB are complementary which means their sum is equal to 90∘.

Let ∠ADB=x∘ and ∠ACB=(90−x) ∘

It is given that BD = 4 m and BC = 9m which means DC = 9 - 4 = 5 m

In △ABC

=tan(90−x)

⇒=cotx       **{tan(90-x) = cot x}**

⇒AB=9cotx             **(1)**

In △ABD

=tanx

⇒=tanx

⇒AB=4tanx                        **(2)**

From **(1) and (2)** , we get

**9cot x=4tan x**

⇒tan2x=

⇒tanx=              **(3)**

Putting equation **(3)**  in equation **(2)** , we get AB=4×3/2=6 **m**

**Therefore, Height of the Tower is equal to 6 m**

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| 4.5 Class work Problems |

**Q1.** Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

**Q2.** As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse,

find the distance between the two ships.

**Q3.** A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

**Q4.** A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.

**Q5.** A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30∘ with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

**Q6.** A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°

**Q7.** A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

**Q8.** A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

**Q9.** A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

**Q10.** From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the

tower.

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| 4.6 Home assignment |

**Q1.** From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

**Q2.** A man is walking along a straight road. He notices the top of a tower subtending an angle A = 60o with the ground at the point where he is standing. If the height of the tower is h = 20 m, then what is the distance (in metres) of the man from the tower?

**Q3.** A little boy is flying a kite. The string of the kite makes an angle of 30o with the ground. If the height of the kite is h = 18m, find the length (in metres) of the string that the boy has used.

**Q4.** Two towers face each other separated by a distance d = 15 m. As seen from the top of the first tower, the angle of depression of the second tower's base is 60o and that of the top is 30o. What is the height (in metres) of the second tower?

**Q5.** A ship of height h = 21 m is sighted from a lighthouse. From the top of the lighthouse, the angle of depression to the top of the mast and the base of the ship equal 30o and 45o respectively. How far is the ship from the lighthouse (in metres)?

**Q6.** Two men on opposite sides of a TV

tower of height 32 m notice the angle of

elevation of the top of this tower to be 45o and 60o respectively. Find the distance (in metres) between the two men.

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| 4.7 Additional problems |

**Q1.** A little boy is flying a kite. The string of the kite makes an angle of 30o with the ground. If the height of the kite is h = 9 m, find the length (in metres) of the string that the boy has used.

**Q2.** A ship of height h = 24 m is sighted

from a lighthouse. From the top of thelighthouse, the angle of depression to the top of the mast and the base of the ship equal 30o and 45o respectively. How far is the ship from the lighthouse (in metres)?

**Q3.** Two men on opposite sides of a TV tower of height 24 m notice the angle of elevation of the top of this tower to be 45o and 60o respectively. Find the distance (in metres) between the two men.

**Q4.** Two men on the same side of a tall building notice the angle of elevation to the top of the building to be 30o and 60o respectively. If the height of the building is known to be h =120 m, find the distance (in metres) between the two men.

**Q5.** A pole of height h = 40 ft has a shadow of length l = 40.00 ft at a particular instant of time. Find the angle of elevation (in degrees) of the sun at this point of time.

**Q6.** You are stationed at a radar base and

you observe an unidentified plane at an altitude h = 2000 m flying towards your radar base at an angle of elevation = 30o. After exactly one minute, your radar sweep reveals that the plane is now at an angle of elevation = 60o maintaining the same altitude. What is the speed (in m/s) of the plane?

**Q7.** Two towers face each other separated by a distance d = 45 m. As seen from the top of the first tower, the angle of depression of the second tower's base is 60o and that of the top is 30o. What is the height (in metres) of the second tower?

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| 4.8 Self Assessment 1: |

**Q1.** A man standing at a point P is watching the top of a tower, which makes an angle of elevation of 300 with the man's eye. The man walks some distance towards the tower to watch its top and the angle of the elevation becomes 600. What is the distance between the base of the tower and the point P?

(a) 43 units

(b) 8 units

(c) 12 units

(d) Data inadequate

**Q2.** From a point P on a level ground, the angle of elevation of the top tower is 300. If the tower is 100 m high, the distance of point P from the foot of the tower is:

(a) 149 m

(b) 156m

(c) 173m

(d) 200

**Q3.** The angle of elevation of a ladder leaning against a wall is 60º and the foot of the ladder is 4.6 m away from the wall. The length of the ladder is:

(a) 2.3m

(b) 4.6m

(c) 7.8m

(d) 9.2m

**Q4.** The angle of elevation of the sun, when the length of the shadow of a tree 3 times the height of the tree, is:

(a) 300

(b) 450

(c) 600

(d) 900

**Q5.** An observer 1.6 m tall is 203 away from a tower. The angle of elevation from his eye to the top of the tower is 300. The heights of the tower is:

(a) 21.6m

(b) 23.2m

(c) 24.72m

(d) None of these

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| 4.9 Self Assessment 2: |

**Q1.** Two men on opposite sides of a TV tower of height 32 m notice the angle of elevation of the top of this tower to be 45o and 60o respectively. Find the distance (in metres) between the two men.

**Q2.** Two men on the same side of a tall building notice the angle of elevation to the top of the building to be 30o and 60o respectively. If the height of the building is known to be *h* =60 m, find the distance (in metres) between the two men.

**Q3.** You are stationed at a radar base and you observe an unidentified plane at an altitude *h* = 3000 m flying towards your radar base at an angle of elevation = 30o. After exactly one minute, your radar sweep reveals that the plane is now at an angle of elevation = 60o maintaining the same altitude. What is the speed (in m/s) of the plane?

**Q4.** A pole of height *h* = 40 ft has a shadow of length *l* = 23.09 ft at a particular instant of time. Find the angle of elevation (in degrees) of the sun at this point of time.

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**Q5.** From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45∘ and 60∘ respectively. Find the height of the tower.

**Chapter 5**

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| Blood Relations |

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| 5.1 Learning objective |

This chapter will introduce you to the proper definitions of common relations; you will learn all the tricks necessary and will also learn how to avoid being tricked, while solving family tree based logical problems.

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| 5.2 Introduction |

The very basics to solve the family tree problems are to know the various relations between the family members. And the actual problem arises when simple relation is defined in a typical way. So to make yourself able to deal with, study the following relations and the manner in which they may be presented.

|  |  |
| --- | --- |
| Mother or Father's son | Brother |
| Mother or Father's Daughter | Sister |
| Mother or Father's Brother | Uncle |
| Mother or Father's sister | Aunt |
| Mother or Father's father | Grandfather |
| Mother or Father's mother | Grandmother |
| Son's wife | Daughter in law |
| Daughter's husband | Son in law |
| Husband's or wife's sister | Sister in law |
| Husband's or wife's brother | Brother in law |
| Brother's son | Nephew |
| Brother's daughter | Niece |
| Uncle or Aunt's son or daughter | Cousin |
| Sister's husband | Brother in law |
| Brother's wife | Sister in law |
| Grandson's or granddaughter’s son or daughter | Great grandson or Great  grand daughter |

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| 5.3 Procedure |

Blood relation problems can be solved using family tree diagrams.

When picturing the family tree,

(i)Differentiate the genders by different symbols

(ii) Identify the generation of people to which they belong.

(iii) Relate people by their relationships in figure.

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| 5.4 Worked out examples |

**Directions for Questions (Q1-Q3):**

Read the following information & answer the questions given below it.

A is the father of C, But C is not his Son

E is the daughter of C, F is the spouse of A.

B is the brother of C, D is the son of B.

G is the Spouse of B, H is the father of G.

**Q1.** Who is the grandmother of D?

(a) A

(b) C

(c) F

(d) H

**Q2**. Who is the son of F?

(a) B

(b) C

(c) D

(d) E

**Q3.** How many number of males are there in the family?

(a) 3

(b) 4

(c) Cannot be determined

(d) None of these

Correct options are

**Q1**: (c)

**Q2:** (a)

**Q3:** (b)

**Explanatory answer:**

From the given information lets draw a family tree diagram.

Let be used to indicate male gender &

be used to indicate female gender.

B

A

H

D

From the figure, the grandmother of D is F

∴ Ans of 1 is (c) F

The son of F is B

∴ Ans of 2 is (a) B

We have used to indicate male gender. From the family tree diagram, there are 4 males.

∴ Ans of 3 is (b) 4

**Directions for Question Q4:**

Read the following information & answer the questions that follow.

(i) ‘P x Q’ means ‘Q is mother of P’

(ii) ‘P+Q’ means ‘P is brother of Q’

(iii) ‘P-Q’ means ‘P is sister of Q’

(iv) ‘P÷Q’ means ‘Q is father of P’

[BANK P.O]

Which of the following definitely means ‘ X is grandson of D’?

(a) X x H ÷ D

(b) A+X x H ÷D

(c) A-X x H÷D

(d) None of these

Correct option is (d)

**Explanatory answer:**

If X is grandson of D, X should be male

gender. But from options (a), (b) & (c) the gender of X cannot be determined.

So the answer is (d) None of these.

**Q5.** If ‘A$B’ means ‘A is brother of B’ ‘A@B’ means ‘A is wife of B’, ‘A#B’ mean ‘A is daughter of B’ and ‘A!B’ means ‘A is father of B’ then which of the following expressions indicates the relationship ‘ K is father-in-law of H’?

(a) H@J$L#P!K

(b) H@J$P!L#K

(c) H@J$L#K!P

(d) H@P$J!L#K (SBI PO)

Correct option is (c)

**Explanatory answer:**

Option (c) says H is the wife of the brother of the daughter of K, The gender of K is male (from H@J$L#K1P)

∴ K is the father-in-law of H is indicated in option (c)

So, answer is option(c)

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| 5.5 Class Work Problems |

**Q1**. Muthu said, “This girl is the wife of the grandson of my mother.” How can Muthu’s only son Suresh be related to the girl, if Muthu has no siblings?

(a) Father

(b) Grandfather

(c) Father–in–law

(d) Husband

**Q2.** A woman introduces a man as the son of the brother of her mother. How is the man’s father related to the woman?

(a) Son

(b) Nephew

(c) Grandson

(d) Uncle

**Q3.** Prasanna said, “This girl is the wife of the grandson of my mother.” How can Prasanna be related to the girl?

(a) Husband

(b) Father

(c) Father–in–law

(d) Grandfather

**Q4.** If Arun says, “Vimala’s mother is the only daughter of my mother”, how is Arun related to Vimala’s father?

(a) Father

(b) Brother

(c) Grandfather

(d) None of these

**Q5.** Pointing to a man in a photograph, Asha said, “His mother’s only daughter is my mother”. How is Asha related to that man?

(a) Wife

(b) Sister

(c) Niece

(d) Nephew

**Q6.** A woman going with a boy is asked by another woman about the relationship between them. The women replied, “My maternal uncle and the maternal uncle of his maternal uncle is the same.” How is the lady and the boy related?

(a) Mother and Son

(b) Aunt and Nephew

(c) Grandmother and Grandson

(d) None of these

**Q7**. A man said to a lady, “Your mother’s

husband’s sister is my aunt.” How can the

lady be related to the man?

(a) Granddaughter

(b) Mother

(c) Daughter

(d) Sister

**Q8.** If X is the brother of the son of Y’s son, how is X related to Y?

(a) Son

(b) Great grandson

(c) Grandson

(d) Brother

**Q9.** Introducing a man, a woman said, “His wife is the only daughter of my father.” How is that man related to the woman?

(a) Husband

(b) Brother

(c) Father–in–law

(d) Maternal uncle

**Q10**. Showing the man receiving the prize, Saroja said, “He is the brother of my uncle’s daughter.” Who is the man to Saroja?

(a) Cousin

(b) Brother–in–law

(c) Nephew

(d) Uncle

**Q11.** Rita told Mani, “The girl I met yesterday at the beach was the youngest daughter of the brother–in–law of my friend’s mother.” How is the girl related to Rita’s friend?

(a) Daughter

(b) Niece

(c) Friend

(d) Cousin

**Q12.** Pointing to a man in a photograph, a woman said, “His brother’s father is the only son of my paternal grandfather.” How is the woman related to the man in the photograph?

(a) Aunt

(b) Daughter

(c) Grandmother

(d) Sister

**Q13**. Pointing to a gentleman, Deepak said, “His only brother is the father of my daughter’s father.” How is the gentleman related to Deepak?

(a) Brother–in–law

(b) Uncle

(c) Father

(d) Grandfather

**Q14.** Pointing to a photograph, Vipul said, “She is the daughter of my paternal grandfather’s only son.” How is Vipul related to the girl in the photograph?

(a) Brother

(b) Grandson

(c) Cousin

(d) Father

**Q15**. Pointing to an old man, Kunal said, “His son Karthik is my son’s uncle.” How can the old man be related to Kunal?

(a) Uncle

(b) Grandfather

(c) Father

(d) Brother

**Q16.** Rahul told Anand, “Yesterday I defeated the only brother of the daughter of my paternal grandmother.” Whom did Rahul defeat?

(a) Son

(b) Brother

(c) Cousin

(d) Father

**Q17**. If Neena says, “Anita’s father Raman is the only son of my father–in–law Mahipal”, then how is Bindu, who is the sister of Anita, related to Mahipal?

(a) Wife

(b) Daughter

(c) Niece

(d) None of these

**Q18.** Pointing to a man on the stage, Rita said, “He is the brother of the daughter of the wife of my husband.” How is the man on the stage related to Rita?

(a) Husband

(b) Nephew

(c) Son

(d) Cousin

**Q19.** A woman introduces a man as the son of the brother of her mother. How is the man related to the woman?

(a) Son

(b) Grandson

(c) Cousin

(d) Uncle

**Q20.** Pointing towards a person, a man said to a woman, “His mother is the only daughter of your father.” How is the woman related to that person?

(a) Mother

(b) Sister

(c) Daughter

(d) Wife

**Q21.** A girl introduced a boy as the son of the daughter of the father of her uncle. The boy can be the girl’s \_\_\_\_

(a) Son

(b) Brother

(c) Uncle

(d) Nephew

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| 5.6 Home assignment [Question from competitive exams] |

**Q1.** Pointing towards a boy, Veena said, “He is the son of only son of my grandfather” . How is that boy related to Veena?

(a) Uncle

(b) Brother

(c) Cousin

(d) Data Inadequate (Bank Clerical)

**Q2.** Introducing Reena, Monika said , “She is the only daughter of my father’s only daughter.” How is Monika related to Reena?

(a) Aunt

(b) Niece

(c) Data Inadequate

(d) None of these (RBI)

**Q3.** Pointing to a man, a woman said, “His mother is the only daughter of my mother.” How is the woman’s mother related to the man?

(a) Mother

(b) Daughter

(c) Sister

(d) Grandmother (MBA)

**Q4.** Pointing towards Rita, Nikhil said, “I am the only son of her mother’s son.” How is Rita related to Nikhil?

(a) Aunt

(b) Niece

(c) Mother

(d)Cousin (IGNOU)

**Q5**. Pointing to a lady a man said, “The son of her only brother is the brother of my wife.” How is the lady related to the man?

(a) Mother’s Sister

(b) Grandmother

(c) Mother-in-Law

(d) Sister of father-in-Law (IITM)

**Q6**. Pointing to Ketan, Namrata said, “He is the son of my father’s only son.” How is Ketan’s mother related to Namrata?.

(a) Daughter

(b) Aunt

(c) Sister

(d) Sister-in-Law (Bank PO)

**Q7.** Looking at the portrait of a man, Harsh said, “His mother is the wife of my father’s son Brother and sisters I have none.” At whose portrait was Harsh looking?.

(a) His son

(b) His cousin

(c) His uncle

(d) His Nephew (MBA)

**Q8.** If ‘P&Q’ Means ‘P’ is the father of Q’ : 'P#Q’ means ‘P’ is mother of Q’: ‘P\*Q’ means ‘P is sister of Q: then how is D related to N in N#A$B\*D?

(a) Nephew

(b) Grandson

(c) Grand daughter

(d) Data Inadequate (SBI PO)

**Q9.** If ‘P x Q’ means ‘P is the daughter of Q’ ; ‘P+ Q’ means ‘P is the father of Q’ ; ‘P÷Q’ means ‘P is the mother of Q’ and ‘P-Q’ means ‘P is the brother of Q’, then in the expression A÷B+C-E x F, how is A related to F?

(a) Mother

(b) Daughter - in-Law

(c) Sister-in-Law

(d) None of these (SBI PO)

**Q10.** Pointing to the women in the picture Rajiv said, “ Her mother has only one grandchild whose mother is my wife.” How is the woman in the picture related to Rajiv?

(a) Cousin

(b) Wife

(c) Data Inadequate

(d) None of these (Bank PO)

**Directions for Questions (Q11-Q13):**

Read the following information & answer the questions given below it.

In a village of Bastar district in Madhya Pradesh, only two types of people live who belong to a tribal class. The first type is known as class A, while the other is known as class B.

In that village, there is no other type of person except these two. The activities of both types of people are governed be perfectly patterned norms of social behaviour. Each person of the tribe has to obey the norms. They are rigid about this. As far as marriage is concerned, the following norms are to be followed

(A)  The people of class A cannot marry any other member of their own class, though they can marry members of class

(B) After being married, each male member ceases to be a member of that class in which he was born but automatically, he becomes the member of the other class to which his wife belongs.

(C) As far as females are concerned, they remain the members of their own class after being married to the other family.

(D) On his birth, the child automatically becomes the member of his mother's class.

(E) When any male member becomes widower or divorcee, then he again belongs to the group in which he was born.

(F) Nobody can marry more than one person according to social laws. [MAT]

**Q11.** Which of the following marriages is not permissible according to the social laws ?

(a) Any girl of class B marries her mother's brother.

(b) Any widower marries his wife's sister

(c) Any boy of class B marries his father's sister

(d) Any widower born in class A marries his brother's widow.

(e) Any widow marries the former divorced husband of her daughter.

**Q12.** One boy, who was born in class B (boy and his wife both can have married and unmarried brothers) ,

(a) Can have his daughter in Class B

(b) Can have a son-in-law born in class A

(c) Can have his uncle from any class

(d) Can have a divorced son in class B

(e) Can have a daughter-in-law born in

class A

**Q13.** Any class B female can have

 (P) Grandfather born in class A

(Q) Grandmother born in class A

(a) Only (P) can be true

(b) Only (Q) can be true

(c) Either (P) or (Q) can be true

(d) Neither (P) nor (Q) can be true

(e) Both (P) and (Q) can be true

**Q14.** A family comprises seven members namely M,N, O, P, Q, R, and S. Among them four are adults and three are children. Of the three children, only R and S are girls. M and P are brothers and M is a pilot. Q is an airhostess married to one of the brothers and has two children. N is married to P and S is their child. Who is O?

(a) M’s son

(b) R’s father

(c) Q’s daughter

(d) P’s son [JMET]

**Q15.** If C $ D means C is the brother of D, C \* D means C is the mother of D, and

C # D means C is the sister of D, which of the following would mean ‘M is the uncle of N’?

(a) M $ C # N

(b) N # M $ C

(c) M $ C \* N

(d) M # C \* N [JMET]

**Q16.** A party consists of grandmother, father, mother, four sons and their wives and one son and two daughters to each of the sons. How many females are there in all?

(a) 14

(b) 16

(c) 18

(d) 24 [MAT]

**Q17.** Daya has a brother, Ankit. Daya is the son of Chandru. Bimil is Chandru's father. In terms of relationship, how is Bimil related to Ankit?

(a) Son

(b) Grandson

(c) Brother

(d) Grandfather [MAT]

**Q18.** Rahul's mother is the only daughter of Monika's father. How is Monika's husband related to Rahul?

(a) Uncle

(b) Father

(c) Grandfather

(d) Brother [MAT]

**Q19.** Lakshmi and Meena are Rohan's wives. Shalini is Meena's step-daughter. How is Lakshmi related to Shalini?

(a) Sister

(b) Mother-in-Law

(c) Mother

(d) Step-mother [MAT]

**Q20.** A group comprising five persons namely M, N, O, P, and Q includes one professor of civil engineering and one of mechanical engineering. In the group, M and P are unmarried women. No woman is either a chemical or a mechanical or a civil engineer.

There is just one married couple in the group with Q as the husband. N is the brother of O and is neither a chemical engineer nor a mechanical engineer. What is the profession of Q?

(a) Civil Engineer

(b) Chemical Engineer

(c) Mechanical Engineer

(d) Cannot be determined [JMET]

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| 5.7 Self Assessment-1 |

**Directions for Questions (Q1 - Q4):**

Read the given data carefully and solve the questions which are given below   
A is the father of two children B and D, who are of different gender.  C is B’s Spouse. E is of the same sex as D.  
B and C have two children : F, who is the same sex as B, & G, who is the same sex as C. E’s mother, H, who is married to L, is the sister of D’s mother, M. E and E’s spouse. I, have two children, J and K, who are the same sex as I. No persons have married more than once and no children have been born out of wedlock.

The only restrictions on marriage are that marriage to a sibling, to a direct descendant, or to more than one person at the same time is forbidden. [MAT]

**Q1**. F  is

(a) G’s brother

(b) G’s sister

(c) B’s daughter

(d) D’s niece or nephew

(e) the same sex as H

**Q2.** According to the rules, D can marry  
(a) F only

(b) G only

(c) J only

(d) J or K only

(e) F, J or K

**Q3.** If L and H divorced, H could marry  
I.   D only      II.  F     III. D or G

(a) I only

(b) II only

(c) III only

(d) I or II, but not both.

(e) II or III, but not both

**Q4.** If the generation of F and K’s parents and their siblings contains more females than males, which of the following must be true?

(a) There are more females than males in F and K’s generation.

(b) J is male.

(c) A is the same sex as D

(d) K and G are the same sex.

(e) D in H’s nephew.

**Directions for Questions(Q5 - Q9):**

Read the given data carefully and solve the questions which are given below.

There are six persons A, B, C, D , E and F. C is the sister of F. B is the brother of E's husband. D is the father of A and grandfather of F. There are two fathers, three brothers and a mother in the group.

[MAT]

**Q5.** Who is the mother ?

(a) A

(b) B

(c) D

(d) E

**Q6.** Who is E's husband ?

(a) B

(b) C

(c) A

(d) F

**Q7.** How many male members are there in the group ?

(a) One

(b) Two

(c) Three

(d) Four

**Q8.** How is F related to E ?

(a) Uncle

(b) Husband

(c) Son

(d) Daughter

**Q9.** Which of the following is a group of brothers ?

(a) ABF

(b) ABD

(c) BFC

(d) BDF

**Directions for Questions (Q10 – Q13):**

Read the given data carefully and solve the questions which are given below.   
Prashant Arora has three children -- Sangeeta, Vimal and Ashish. Ashish married to Monika, the eldest daughter of Mr. and Mrs. Roy. The Roys married their youngest daughter to the eldest son of Mr. and Mrs. Sharma, and they had two children named Amit and Shashi. The Roys have two more children, Roshan and Vandana, both elder to Veena. Sameer and Ajay are sons of Ashish and Monika. Rashmi is the daughter of Amit. [MAT]

**Q10.** What is the surname of Rashmi ?

(a) Sharma

(b) Roy

(c) Arora

(d) cannot be determined

(e) None of these

**Q11.** How is Sameer related to the father of Monika ?

(a) Grandson

(b) Son

(c) Cousin

(d) Son-in-law

(e) None of these

**Q12.** How is Mrs. Roy related to Ashish ?

(a) Aunt

(b) Mother-in-law

(c) Mother

(d) Sister-in-law

(e) None of these

**Q13.** What is the surname of Sameer ?

(a) Roy

(b) Sharma

(c) Arora

(d) Cannot be determined

(e) None of these

**Q14**. X-Z means X is the mother of Z; X\* Z means X is the father of Z ; X+Z means X is daughter of Z. Now if M-N\*T+Q then which of the following is false

(a) T is N’s daughter

(b) M is mother in law of N

(c) Q is wife of N

(d) T is granddaughter of M

**Q15.** P+ Q means P is brother of Q.; P\* Q means P is father of Q ;P-Q means P is sister of Q. Which of the following mean S is the niece of T?

(a) T \*M+S-K

(b) K-S\*M+T

(c) T+M\*S-K

(d) T\*S+M-K

(e) None of these

**Q16.** P\*Q means P is sister of Q.P+Q means P is father of Q;P-Q means P is mother of Q. Which of the following means S is aunt of T?

(a) T\*M+S

(b) S+T\*M

(c) S\*M+T

(d) S\*M+R-T

(e) None of these.

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| 5.8 Self Assessment 2 |

**Q1.** Pointing to a photograph, Anu said “He is the son of the father of my sister’s brother.” How is the person in the photo related to Anu?

(a) Mother

(b) Father

(c) Maternal Uncle

(d) Cousin

(e) None of these

**Q2.** Pointing to a photograph, a man said “I have no brother or sister but that man’s father is my father’s son.” Whose photograph was it?

(a) His own

(b) His son’s

(c) His father’s

(d) His nephew’s

(e) None of these

**Q3.** Looking at a portrait of a man, Harshan said “His wife is the wife of my father’s son. I have no Brothers and sisters”. Whose photograph was it?

(a) His son

(b) His cousin

(c) His uncle

(d) His nephew

(e) None of these

**Q4.** Pointing to a photograph a lady tells Pramod, “I am the only daughter of this lady and her son is your maternal uncle” How is the speaker related to Pramod’s father?

(a) Sister in law

(b) Wife

(c) Either a or b

(d) neither a nor b

**Q5.** Pointing to a photograph a man says ‘The lady in the photograph is my nephew’s maternal grandmother.” How is the lady in the photograph related to the man’s sister who has no other siblings?

(a) Cousin

(b) Sister in law

(c) Mother

(d) Mother in law

**Q6.** A and B are brothers. C and D are sisters. A’s son is D’s brother. How is B related to C?

(a) Father

(b) Brother

(c) Grand Father

(d) Uncle

(e) None of these

**Q7.** Dharshan has a brother Anil. Dharshan is the son of Chandra Bimil is Chandra’s father. How is Anil related to

Bimil ?

(a) Son

(b) Grandson

(c) Brother

(d) Grandfather

(e) None of these

**Q8.** Deepak is brother of Ravi and Rekha is sister of Atul. Ravi is son of Rekha.How is Deepak related to Rekha?

(a) Son

(b) Brother

(c) Nephew

(d) Father

**Q9.** A is father of C and D is son of B. E is brother of A. If C is sister of D, how is B related to E

(a) Daughter

(b) Brother in law

(c) Husband

(d) Sister in law

(e) None of these

**Directions for Questions (Q10 -Q12):**

Read the given data carefully and solve the questions follow.

(i)  A, B, C, D, E and F are travelling in a bus.

(ii) There are two reporters, two technicians, one photographer and one writer with in the given group.  
(iii) The photographer A is married to D who is also said to be one of the reporters.

(iv) The writer is married to B who is of the same profession as that of F.

(v) A, C, B, D are two married couples and nobody in the group has same profession.

(vi) F is brother of C.

**Q10.** Which of the following pair is a couple?

(a) AB

(b) BC

(c) BD

(d) AE

**Q11.** How is C related to F ?

(a) Brother

(b) Sister

(c) Uncle

(d) Cannot be determined

**Q12.** Which of the following is a pair of reporters?

(a) AB

(b) CE

(c) DE

(d) DF

**Chapter 6**

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| Arrangements, Sequencing and Scheduling |

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| 6.1 Learning objective: |

This chapter will introduce you to arrange items or things or persons in an order as per the given constraints.

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| 6.2 Introduction |

The information is given about certain number of things, which have to be arranged in rows or matrix form.

Sometimes this involves scheduling.

Arrangement might be linear or circular arrangement. It could be arranging people or items in a row or around a table satisfying the given conditions.

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| 6.3 Procedure |

1. Read the information clearly

2. Picturise the information clearly.

(Represent the given information in a more understandable way.)

3. Arrange the items satisfying the given conditions.

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| 6.4 Quiz |

**Directions for Questions (Q 1- Q3):**

Read the following information and answer the questions given below it:

(i) There are three houses on each side of the road.

(ii) These six houses are labeled as P, Q, R, S, T and U.

(iii) The houses are of different colours,

namely, Red, Blue, Green, Orange, Yellow and White.

(iv) The houses are of different heights.

(v) T, the tallest house, is exactly opposite to the Red coloured house.

(vi) The shortest house is exactly opposite to the Green coloured house.

(vii) U, the Orange coloured house, is located between P and S.

(viii)R, the Yellow coloured house, is exactly opposite to P.

(ix) Q, the Green coloured house, is exactly opposite to U.

(x) P, the White coloured house, is taller than R, but shorter than S and Q. [CAT]

**Q1.** What is the colour of the tallest house?

**Q2.** What is the colour of the housediagonally opposite to the Yellow coloured house?

**Q3.** Which is the second tallest house?

**Directions for Questions(Q4 & Q5):**

Read the following information and answer the questions given below it:

Five horses, Red, White, Grey, Black and Spotted participated in a race. As per the rules of the race, the persons betting on the winning horse get four times the bet amount and those betting on the horse that came in second get thrice the bet amount. Moreover, the bet amount is returned to those betting on the horse that came in third, and the rest lose the bet amount. Raju bets Rs. 3000, Rs. 2000 Rs. 1000 on Red, White and Black horses respectively and ends up with no profit and no loss. [CAT]

**Q4.**Which of the following cannot be true?

(a) At least two horses finished before Spotted

(b) Red finished last

(c) There were three horses between Black and Spotted

(d) There were three horses between White and Red

(e) Grey came in second

**Q5.** Suppose, in addition, it is known that Grey came in fourth. Then which of the following cannot be true?

(a) Spotted came in first

(b) Red finished last

(c) White came in second

(d) Black came in second

(e) There was one horse between Black and White

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| 6.5 Worked out examples |

**Directions for Questions (Q1 - Q5):**

Read the following information and answer the questions given below it:

There are five friends Sachin, Kunal, Mohit, Anuj and Rohan. Sachin is shorter than Kunal but taller than Rohan. Mohit is tallest. Anuj is a little shorter than Kunal and little taller than Sachin.

**Q1.** Who is the shortest?

(a) Rohan

(b) Sachin

(c) Anuj

(d) Kunal

(e) None of these

**Q2.** If they stand in the order of their heights, who will be in the middle?

(a) Kunal

(b) Rohan

(c) Sachin

(d) Anuj

(e) None of these

**Q3.** If they stand in the order of increasing heights, who will be the second?

(a) Anuj

(b) Sachin

(c) Rohan

(d) Kunal

(e) None of these

**Q4.** Who is the second tallest?

(a) Sachin

(b) Kunal

(c) Anuj

(d) Rohan

(e) None of these

**Q5.** Who is taller than Anuj but shorter than Mohit?

(a) Kunal

(b) Rohan

(c) Sachin

(d) Date Inadequate

(e) None

Correct option are

**Q1** (a) **Q2** (d)

**Q3** (b) **Q4**(b)

**Q5** (a)

**Explanatory Answer:**

From the given information

We could identify Mohit is the tallest person in the group, second tallest person is Kunal & the next taller person is Anuj & fourth taller person is Sachin & the least tall (shortest) person is Rohan.

1. The shortest person is Rohan

2. Anuj would stand middle if they stand in the order of height.

3. If the stand in the order of increasing freights, Sachin would be the second person.

4. The second tallest person is Kunal.

5. Kunal is taller than Anuj and shorter then Mohit.

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| 6.6 Class work Problems |

**Q1.** A team of four is to be selected from eight persons - A, B, C, D, E, F, G and H such that

if A is selected, then B is not selected,

if B is selected then C is not selected,

if C is selected then D is not selected

if D is selected then E is not selected

if E is selected then F is not selected,

if F is selected then G is not selected and

if G is selected then H is not selected.

ln how many ways can the team be selected?

(a) 4

(b) 6

(c) B

(d) None of these

**Directions for Questions (Q2-Q4):**

Read the given passage and answer the questions:

Six products, namely U, V, W, X, Y, and Z are to be placed in the display windows of a shop. There are six display windows numbered 1, 2, 3, 4, 5, and 6 from left to right in such a way that one product is showcased in one window only. However, U cannot be placed adjacent to V. W must be immediately to the left of X and Z cannot be in window number 6. [JMET]

Note: The direction (left or right) should be determined with respect to the observer /shopper.

**Q2.** Which of the following products CANNOT be placed in window number 1?

(a) U

(b) V

(c) W

(d) X

**Q3.** If X is placed in window number 3, in which window can W be placed?

(a) 1

(b) 2

(c) 4

(d) 5

**Q4.** If U is placed in window number 5, which of the following products must be placed in window number 6?

(a) V

(b) W

(c) Y

(d) X

**Q5.** The Chinese are shorter than the Italians; the Americans are taller than the French; the French are taller than the Chinese.

From the information given above, which one of the following statement can be logically inferred?

(a) The Italians are shorter than the Chinese

(b) The French are taller than the Americans

(c) The Chinese are taller than the French

(d) The Americans are taller than the Chinese [JMET]

**Q6.** Ram, Qadir, Sorabji, Charles and Gurdeep all roll the same unbiased dice one after the other and note down their respective numbers. Ram gets an even number; Qadir gets a number greater than that of Gurdeep; Sorabji gets the same number as Charles and the addition of Charles’ and Ram’s numbers is odd. If Qadir’s number is 4, what is the maximum of the addition of the numbers obtained by all five?

(a) 22

(b) 23

(c) 25

(d) 27 [JMET]

**Directions for Questions (Q7 and Q8):**

Read the given passage and answer the questions:

Nine shops in a multiplex are connected by corridors. Anyone visiting these shops must begin at the reception which is part of shop number 1. From there, the other shops may be visited via different corridors connecting them. The details of the corridors connecting various shops are as under:

(i) Corridors connect the reception to shops, 2, 3 and 6.

(ii) A corridor connects the reception to shop 2 and then to shop 4.

(iii) Corridors connect shops 2 and 3 to shop 7.

(iv) Shop 6 is connected to shops 3, 5, and 9.

(v) Shop 7 is connected to shop 9.

(vi) Shops 7 and 9 are connected to

shop 8.[JMET]

**Q7.** Without visiting any shop more than once, what is the MAXIMUM number of shops a person can visit?

(a) 5

(b) 6

(c) 7

(d) 8

**Q8.** If a visitor wants to travel to shops 4 and 5, which one of the following must be FALSE?

(a) The visitor will go to every shop except shop 3

(b) The visitor will go to shop 2 only once

(c) The visitor will not visit any shop more than once

(d) The visitor will visit at least six different shops

**Q9.** Harry is younger than Latif; Mahesh is of the same age as Latif; Randeep is exactly five years older than Mahesh, and Sonam is younger than Randeep but older than Harry by at the most four years. What is the minimum difference between the ages of Randeep and Harry? Assume that ages of all persons in this question are integer values.

(a) four years

(b) five years

(c) six years

(d) eight years [JMET]

**Directions for Questions (Q10 - Q12):**

Read the given passage and answer the questions:

Any additional information provided with a particular question pertains to that individual question only.

Eight entrepreneurs (J, K, L, M, N, O, P, Q) are selected for excellence awards for their outstanding contributions to business. Besides nurturing their own industries, two of these entrepreneurs, viz., J and L, also patronize the telecom industry while two others, viz., M and P, also patronize the computer industry. In arranging the seats, it was decided that the entrepreneurs who patronize other industries in addition to their own should not be seated together. [JMET]

**Q10.** Which one of the following combinations is possible in the seating arrangements?

(a) KMLNJ

(b) JPQOL

(c) JKNM

(d) JOLPQ

**Q11.** In order to ensure a proper seating arrangement, M should sit between:

(a) N and O

(b) K and J

(c) L and N

(d) O and P

**Q12.** Which one of the following can be seated next to O?

(a) J only

(b) Q only

(c) K only

(d) Any of the above

**Directions for Questions (Q13 - Q16):**

Answer the following questions based on the information given below:

In a sports event, six teams (A, B, C, D, E and F) are competing against each other. Matches are scheduled in two stages. Each team plays three matches in Stage-I and two matches in Stage-II. No team plays against the same team more than once in the event. No ties are permitted in any of the matches. The observations after the completion of Stage-I and Stage-II are as given below.

Stage-I:

One team won all the three matches.

Two teams lost all the matches.

D lost to A but won against C and F.

E lost to B but won against C and F.

B lost at least one match.

F did not play against the top team of Stage-I.

Stage-II:

The leader of Stage-I lost the next two matches.

Of the two teams at the bottom after Stage-I, one team won both matches, while the other lost both matches.

One more team lost both matches in Stage-II. [CAT]

**Q13.** The team(s) with the most wins in the event is (are) :

(a) A

(b) A & C

(c) F

(d) E

(e) B & E

**Q14.** The two teams that defeated the leader of Stage-I are:

(a) F & D

(b) E & F

(c) B & D

(d) E & D

(e) F & D

**Q15.** The only team(s) that won both the matches in Stage-II is (are):

(a) B

(b) E & F

(c) A, E & F

(d) B, E & F

(e) B & F

**Q16.** The teams that won exactly two matches in the event are:

(a) A, D & F

(b) D & E

(c) E & F

(d) D, E & F

(e) D & F

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| 6.7 Home assignment Problems [Questions from competitive exams] |

**Directions for Questions (Q1 – Q4):**

Read the given passage and answer the questions:

Help Distress (HD) is an NGO involved in providing assistance to people suffering from natural disasters. Currently, it has 37 volunteers.

They are involved in three projects: Tsunami Relief (TR) in Tamil Nadu, Flood Relief (FR) in Maharashtra, and Earthquake Relief (ER) in Gujarat.

Each volunteer working with Help Distress has to be involved in at least one relief work project.

A Maximum number of volunteers are involved in the FR project.

Among them, the number of volunteers involved in FR project alone is equal to the volunteers having additional involvement in the ER project.

The number of volunteers involved in the ER project alone is double the number of volunteers involved in all the three projects.

17 volunteers are involved in the TR project.

The number of volunteers involved in the TR project alone is one less than the number of volunteers involved in ER project alone.

Ten volunteers involved in the TR project are also involved in at least one more project. [CAT]

**Q1.** Based on the information given above, the minimum number of volunteers involved in both FR and TR projects, but not in the ER project is:

(a) 1

(b) 3

(c) 4

(d) 5

**Q2.** Which of the following additional information would enable to find the exact number of volunteers involved in various projects?

(a) Twenty volunteers are involved in FR.

(b) Four volunteers are involved in all the three projects.

(c) Twenty three volunteers are involved in exactly one project.

(d) No need for any additional information.

**Q3.** After some time, the volunteers who were involved in all the three projects were asked to withdraw from one project. As a result, one of the volunteers opted out of the TR project, and one opted out of the ER project, while the remaining ones involved in all the three projects opted out of the FR project. Which of the following statements, then, necessarily follows?

(a) The lowest number of volunteers is now in TR project.

(b) More volunteers are now in FR project as compared to ER project.

(c) More volunteers are now in TR project as compared to ER project.

(d) None of the above.

**Q4.** After the withdrawal of volunteers, as indicated in Q3, some newvolunteers joined the NGO. Each one of them was allotted only one project in a manner such that, the number of volunteers working in one project alone for each of the three projects became identical. At that point, it was also found that the number of volunteers involved in FR and ER projects was the same as the number of volunteers involved in TR and ER projects. Which of the projects now has the highest number of volunteers?

(a) ER

(b) FR

(c) TR

(d) Cannot be determined

**Directions for Questions (Q5 – Q8):**

Read the given passage and answer the questions:

U, V, X, Y and Z collected CDs of yesteryears’ melodious songs. They collected a total of 100 CDs. None of them have collected less than 10 CDs. No two among them collected the same number of CDs. Also,

(i) U collected the same number of CDs as

V and X put together.

(ii) X collected 3 more CDs than the cube of an integer.

(iii) The number of CDs collected by U was the square of an integer.

(iv) The number of CDs collected by V was either the square or the cube of an integer.

(v) The number of CDs collected by Y and Z are in the ratio 4 : 3. [JMET]

**Q5.** How many of the individual collection(s) of CDs was/were prime numbers?

(a) 0

(b) 1

(c) 2

(d) 3

**Q6.** What was the number of CDs collected by U?

(a) 19

(b) 36

(c) 52

(d) 64

**Q7.** What was the number of CDs collected by V?

(a) 16

(b) 25

(c) 46

(d) 64

**Q8.** What was the difference in the number of CDs collected by X and Y?

(a) 5

(b) 7

(c) 9

(d) 11

**Directions for Questions (Q9 - Q12):**

Answer the questions on the basis of the information given below.

Twenty one participants from four continents (Africa, Americas, Australia, and Europe) attended a United Nations conference. Each participant was an expert in one of four fields, labour, health, population studies, and refugee relocation. The following five facts about the participants are given.

(i) The number of labour experts in the camp was exactly half the number of experts in each of the three other categories.

(ii) Africa did not send any labour expert. Otherwise, every continent, including Africa, sent at least one expert for each category.

(iii) None of the continents sent more than three experts in any category.

(iv) If there had been one less Australian expert, then the Americas would have had twice as many experts as each of the other continents.

(v) Mike and Alfanso are leading experts of population studies who attended the conference. They are from Australia.

[CAT]

**Q9.** Alex, an American expert in refugee relocation, was the first keynote speaker in the conference. What can be inferred about the number of American experts in refugee relocation in the conference, excluding Alex?

i. At least one ii. At most two

(a) Only i and not ii

(b) Only ii and not I

(c) Both i and ii

(d) Neither i nor ii

**Q10.** Which of the following numbers cannot be determined from the information given?

(a) Number of the labour experts from the Americas.

(b) Number of the health experts from Europe.

(c) Number of the health experts from Australasia.

(d) Number of the experts in refugee relocation from Africa.

**Q11.** Which of the following combinations is NOT possible?

(a) 2 experts in population studies from the Americas and 2 health experts from Africa attended the conference.

(b) 2 experts in population studies from the Americas and 1 health expert from Africa attended the conference.

(c) 3 experts in refugee relocation from the Americas and 1 health expert from Africa attended the conference.

(d) Africa and America each had 1 expert in population studies attending the conference.

**Q12.** If Ramos is the lone American expert in population studies, which of the following is NOT true about the numbers of experts in the conference from the four continents?

(a) There is one expert in health from Africa.

(b) There is one expert in refugee relocation from Africa.

(c) There are two experts in health from the Americas.

(d) There are three experts in refugee relocation from the Americas.

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| 6.8 Self Assessment 1 |

**Directions for Questions (Q1 – Q4):**

Answer these questions based on the information given.

In a car exhibition, seven cars of seven different companies viz. Cadillac, Ambassador, Fiat, Maruti, Mercedes, Bedford and Fargo were displayed in a row, facing east such that:

I. Cadillac car was to the immediate right of Fargo.

II. Fargo was fourth to the right of Fiat.

III. Maruti car was between Ambassador and Bedford.

IV. Fiat, which was third to the left of Ambassador car, was at one of the ends.

[MAT]

**Q1.** Which of the following was the correct position of the Mercedes?

(a) Immediate right of Fiat

(b) Immediate left of Bedford

(c) Between Bedford and Fargo

(d) Fourth to the right of Maruti

**Q2.** Which of the following is definitely true?

(a) Fargo car is between Ambassador and Fiat.

(b) Cadillac car is to the immediate left of Mercedes.

(c) Fargo is to the immediate right of Cadillac.

(d) Maruti is the fourth to the right of Mercedes.

**Q3.** Which of the following is definitely true?

(a) Maruti is to the immediate left of Ambassador.

(b) Bedford is to the immediate left of Fiat.

(c) Bedford is at one of the ends.

(d) Fiat is second to the right of Maruti.

**Q4.** Which of the following groups of cars is to the right of the Ambassador car?

(a) Cadillac, Fargo and Maruti

(b) Maruti, Bedford and Fiat

(c) Mercedes, Cadillac and Fargo

(d) Bedford, Cadillac and Fargo

**Q5.** A team of four is to be selected from seven persons. A, B, C, D, E, F and G suchthat, at most one of A and B can be selected, at most one of C and D can be selected, at most one of E and F can be selected, at least one of G and C must not be selected and at least one of B and E must not be selected. ln how many ways can the team be selected?

(a) 5

(b) 4

(c) 3

(d) 2

**Directions for Questions (Q6 - Q10):**

Answer these questions based on the information given.

A team of four out of seven girls - Akila, Sangavi, Karthika, Kavya, Neethika, Saranya and Vidya is to be selected under the following constraints.

(i) Kavya and Saranya should not be selected together.

(ii) Only if Karthika is selected, Akila will be selected.

(iii) At most one of Neethika and Saranya must be selected.

(iv) At most one of Sangavi and Vidya must be selected.

(v) At least one of Akila and Saranya must be selected.

**Q6.** Who must always be selected?

(a) Karthika

(b) Kavya

(c) Saranya

(d) Neethika

**Q7.** If Vidya must be selected then in how many ways the team can be selected?

(a) 0

(b) I

(c) 2

(d) 3

**Q8.** If Kavya is not to be selected, then which of the following cannot be a possible team?

(a) Akila, Karthika, Saranya, Vidya

(b) Akila, Sangavi, Karthika, Neethika

(c) Akila, Karthika, Neethika, Vidya

(d) Akila, Sangavi, Karthika, Vidya

**Q9.** lf Saranya is not to be selected, then in how many ways the team can be selected?

(a) 5

(b) 6

(c) 3

(d) 4

**Q10.** If Saranya and Vidya are selected, then the other two must be

(a) Akila and Karthika

(b) Karthika and Neethika

(c) Karthika and Kavya

(d) Akila and Sangavi

**Directions for Questions (Q11 - Q15):**

Read the following information and answer the questions given below it:

J, K, L, M and N are five boys in a class. They are ranked in the order of heights - from the tallest to the shortest - and in order of cleverness - from the cleverest to the dullest. K is taller than N, but not as clever as J and L, whereas M is the cleverest of all but shorter than J. While L is shorter than M but taller than K, L is not as clever as J. No two persons got the same ranks in any of these parameters.

**Q11.** Who is the third in the order of heights?

(a) J

(b) N

(c) K

(d) L

**Q12.** lf N is not the last in at least one of the two comparisons. which of the following is the dullest of all the five?

(a) K

(b) L

(c) M

(d) J

**Q13.** lf L is the third in order of cleverness, who is the dullest of all?

(a) M

(b) N

(c) J

(d) Cannot be determined

**Q14.** Who among the following is cleverer as well as taller than K?

(a) L and J only

(b) N

(c) L and N

(d) J, L and M

**Q15.** How many people are definitely shorter than K?

(a) I

(b) 2

(c) 4

(d) 3

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| 6.9 Self Assessment 2 |

**Directions for Questions (Q1 – Q4):**

Answer these questions based on the information given.

The year is 2089, Beijing, London, New York, and Paris are in contention to host the 2096 Olympics. The eventual winner is determined through several rounds of voting by members of the IOC with each member representing a different city. All the four cities in contention are also represented in IOC. In any round of voting, the city receiving the lowest number of votes in that round gets eliminated. The survivor after the last round of voting gets to host the event.

A member is allowed to cast votes for at most two different cities in all rounds of voting combined. (Hence, a member becomes ineligible to cast a vote in a given round if both the cities (s) he voted for in earlier rounds are out of contention in that round of voting.)

A member is also ineligible to cast a vote in a round if the city (s) he represents is in contention in that round of voting.

As long as the member is eligible, (s) he must vote and vote for only one candidate city in any round of voting.

The following incomplete table shows the information on cities that received the maximum and minimum votes in different rounds, the number of votes cast in their favour, and the total votes that were cast in those rounds.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Round | Total  Votes cast | Maximum  Votes cast | | Eliminated | |
| city | votes | City | votes |
| 1 |  | London | 30 | New York | 12 |
| 2 | 83 | Paris | 32 | Bejing | 21 |
| 3 | 75 |  |  |  |  |

It is also known that:

All those who voted for London and Paris in round 1, continued to vote for the same cities in subsequent rounds as long as these cities were in contention. 75% of those who voted or Beijing in round 1, voted for Beijing in round 2 as well.

Those who voted for New York in round 1, voted either for Beijing or Paris in round 2.

The difference in votes cast for the two contending cities in the last round was 1.

50% of those who voted for Beijing in round 1, voted for Paris in round 3. [CAT]

**Q1.** What percentage of members from among those who voted for New York in round 1, voted for Beijing in round 2?

(a) 33.33

(b) 50

(c) 66.67

(d) 75

**Q2.** What is the number of votes cast for Paris in round 1?

(a) 16

(b) 18

(c) 22

(d) 24

**Q3.** Which of the following statements must be true?

(I) IOC member from New York must have voted for Paris in round 2.

(II) IOC member from Beijing voted for London in round 3.

(a) Only I

(b) Only II

(c) Both I and II

(d) Neither I nor II

**Q4.** What percentage of members from among those who voted for Beijing in round 2 and were eligible to vote in round 3, voted for London?

(a) 33.33

(b) 38.10

(c) 50

(d) 66.67

**Directions for Questions (Q5 – Q8):**

Answer these questions based on the information given.

In the table below is the listing of players, seeded from highest (#1) to lowest (#32), who are due to play in an Association of Tennis Players (ATP) tournament for women. This tournament has four knockout rounds before the final, i.e., first round, second round, quarterfinals, and semifinals.

In the first round, the highest seeded player plays the lowest seeded player (seed # 32) which is designated match No. 1 of first round; the 2nd seeded player plays the 31st seeded player which is designated match No. 2 of the first round, and so on. Thus, for instance, match No. 16 of first round is to be played between 16th seeded player and the 17th seeded player. In the second round, the winner of match No. 1 of first round plays the winner of match No. 16 of first round and is designated match No. 1 of second round Similarly, the winner of match No. 2 of first round plays the winner of match No. 15 of first round, and is designated match No. 2 of second round. Thus, for instance, match No. 8 of the second round is to be played between the winner of match No. 8 of first round and the winner of match No. 9 of first round. The same pattern is followed for later rounds as well.

Seed # Name of Player

1. Maria Sharapova

2. Lindsay Davenport

3. Amelie Mauresmo

4. Kim Clijsters

5. Svetlana Kuznetsova

6. Elena Dementieva

7 Justine Henin

8 Serena Williams

9 Nadia Petrova

10 Venus Williams

11 Patty Schnyder

12 Mary Pierce

13 Anastasia Myskina

14 Alicia Molik

15 Nathalie Dechy

16 Elena Bovina

17 Jelena Jankovic

18 Ana Ivanovic

19 Vera Zvonareva

20 Elena Likhovtseva

21 Daniela Hantuchova

22 Dinara Safina

23 Silvia Farina Elia

24 Tatiana Golovin

25 Shinobu Asagoe

26 Francesca Schiavone

27 Nicole Vaidisova

28 Gisela Dulko

29 Flavia Pennetta

30 Anna Chakvetadze

31 Ai Sugiyama

32 Anna-lena Groenefeld [CAT]

**Q5.** If there are no upsets (a lower seeded player beating a higher seeded player) in the first round, and only match Nos. 6, 7, and 8 of the second round result in upsets, then who would meet Lindsay Davenport in quarter finals, in case Davenport reaches quarter finals?

(a) Justine Henin

(b) Nadia Petrova

(c) Patty Schnyder

(d) Venus Williams

**Q6.** If the top eight seeds make it to the

quarterfinals, then who, amongst the players listed below, would definitely not play against Maria Sharapova in the final, in case Sharapova reaches the final?

(a) Amelie Mauresmo

(b) Elena Dementieva

(c) Kim Clijsters

(d) Lindsay Davenport

**Q7.** If, in the first round, all even numbered matches (and none of the odd numbered ones) result in upsets, and there are no upsets in the second round, then who could be the lowest seeded player facing Maria Sharapova in semi-finals?

(a) Anastasia Myskina

(b) Flavia Pennetta

(c) Nadia Petrova

(d) Svetlana Kuznetsova

**Q8.** If Elena Dementieva and Serena Williams lose in the second round, while Justine Henin and Nadia Petrova make it to the semi-finals, then who would play Maria Sharapova in the quarterfinals, in the event Sharapova reaches quarterfinals?

(a) Dinara Safina

(b) Justine Henin

(c) Nadia Petrova

(d) Patty Schnyder

**Directions for Questions (Q9 - Q11):**

Read the following information and answer the questions. Any additional information provided with a particular question pertains to that individual question only.

The production, marketing, human resource, finance, and management information system managers of a particular company meet for a round table meeting to discuss the strategy of the company. Out of the five, three are men and the other two are women. The following restrictions apply to their seating arrangement

(i) The two women will not be seated next to each other.

(ii) The production manager, a man, will always be seated as far as possible from the marketing manager.

(iii) The finance manager will always be seated next to a woman. [JMET]

**Q9.** If the human resource manager is a man, which of the following is definitely NOT true?

(a) The marketing manager is sitting in between the two women

(b) The marketing manager is sitting in between two men

(c) The finance manager is a man

(d) The marketing manager is a woman

**Q10.** If the finance manager always has a woman to his right, in how many different arrangements can the marketing manager be a woman?

(a) 2

(b) 3

(c) 4

(d) 5

**Q11.** If the management information system manager is sitting just to the right of the production manager, which of the following statement(s) is/are definitely TRUE?

(i)The marketing manager and the finance manager sit next to each other five times.

(ii)The human resource manager has to be a woman.

(iii) The management information system manager has to be a man.

(a) (i) only

(b) (i) and (ii)

(c) (i) and (iii)

(d) (ii) and (iii)

**Self Study**

**Chapter 7**

|  |
| --- |
| Statistics |

|  |
| --- |
| 7.1 Learning objective: |

In this chapter we will learn averages, deviations, and its interpretations

|  |
| --- |
| 7.2 Averages |

**7.2.1 Arithmetic mean (AM)()**

Given x1,x2,x3,…,xn ( n discrete data)

AM=

*Note:*  = 0

If and are the AMs of two group of data having n1 and n2 data, then their combined AM is

Combined AM =

**7.2.2 Geometric mean (GM)**

Given x1, x2, x3,…,xn ( n positive discrete data)

**7.2.3 Harmonic Mean(HM)**

Given x1, x2, x3,…,xn ( n discrete data, none is zero)

**7.2.4. Median (M)**

The middle most value when arranged in ascending or descending order.

Median does not depend on all the items.

Median divides the data into equal halves.

**7.2.5. Mode (Z)**

It is the most often occurring data.

|  |
| --- |
| 7.3 Measure of dispersion / Deviation |

**7.3.1 Range=maximum values-minimum value**

**7.3.2 Mean Deviation ( MD)**

A=mean/median/mode.

n= number of data.

**7.3.3 Standard Deviation (SD)**

Where x1, x2, x3,…,xn are ndata.

**7.3.4 Coefficient of Variation**

CV =

|  |
| --- |
| 7.4 Worked out examples |

**Q1.** The arithmetic mean of

80,100,120,250,400 is

(a) 100

(b) 120

(c) 190

(d) 200

(e) 210

The correct answer is (c)

**Explanatory answer:**

AM = 190

**Q2.** If the arithmetic mean of x1, x2, x3,…,xn is , then the arithmetic mean of x1+10, x2+10, x3+10,…,xn+10 is

(a)

(b)

(c)

(d)

(e) 0

The correct answer is (c)

**Explanatory answer:**

When all the data are shifted by 10, the average also shifts by 10.

Can you use the formula and get the same result.

**Q3**. If is the mean of first 20 prime numbers p1,p2,p3,…,p20, then

(p1- ) + (p2- )+…+ (pn- )=

(b) -20

(c) 0

(d)

(e) +20

The correct answer is (c)

**Explanatory answer:**

Mean about mean is zero.

**Q4.** The geometric mean of

ab, , (a + b)2 is

(a)

(b) (a+b)

(c) ab

(d) (a+b)2

(e) (a+b)3

**Q5.** What is the HM of 1, ½, 1/3,…,1/n?

(a) n

(b) 2n

(c)

(d)

The correct answer is (c)

**Explanatory answer:**

HM=

**Q6.** If the AM and HM for positive numbers is 5 and 3.2 respectively then the GM will be

(a) 16

(b) 4.10

(c) 4.05

(d) 4

The correct answer is (d)

**Explanatory answer:**

GM2 = AMxHM

GM 2 = 5x3.2

GM = 4

**Q7**. The standard deviation of first 10 odd positive integers is

(a)

(b)

(c)

(d)

(e)

The correct answer is (d)

**Explanatory answer:**

The first 10 odd numbers are 1,3,5,7,9,11,13,15,17,19.

The mean of this data is 10.

**Q8.** The mean deviation, about mean for the data 11, 13, 17, 19, 25, 29 is

(a) 8/3

(b) 10/3

(c) 4

(d) 14/3

(e) 16/3

The correct answer is (e)

**Explanatory answer:**

Mean = (11+13+17+19+25+29)/6

=114/6 =19

**Q9**. The variance of the numbers

-40, -30, -20, -10, 0 is

(a) -200

(b) 50

(c) 100

(d) 150

(e) 200

The correct answer is (e)

**Explanatory answer:**

**Q10**. If the variance of a set of observations is 4096, then the standard deviation is

(a) ± 66

(b) ± 64

(c) ±48

(d) 64

(e) 66

The correct answer is (d)

**Explanatory answer:**

SD is the positive square root of variance.

**Q11.** The standard deviation of 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 12 is

(a)

(b)

(c)

(d)

(e)

The correct answer is (d)

**Explanatory answer:**

|  |
| --- |
| 7.5 Problems for Practice |

**Q1.** Find the arithmetic mean of the numbers a and b if their geometric mean and harmonic mean are 4 and 2.4 respectively.

(a) 5

(b) 8

(c) 10

(d) 25

(e) 50

**Q2.** The arithmetic mean of first 'n' odd positive integers is

(a) n - 1

(b) n

(c)- n + 1

(d) 2n - 1

(e) n2

**Q3.** If x1, x2, x3, …, xn are the observations, then is least when A is

(a) mean

(b) median

(c) mode

(d) either mean or median

(e) either median or mode

**Q4.** The geometric mean of 4, 8, 16, 32, 64 is

(a) 8

(b) 16

(c) 24

(d) 32

(e) 40

**Q5.** The median of first 10 prime numbers is

(a) 11

(b) 12

(c) 13

(d) 17

(e) 19

**Q6.** The standard deviation of -6, -4, -2, 0, 2, 4, 6 is

(a) -4

(b) -2

(c) 0

(d) 2

(e) 4

**Q7.** The geometric mean of 5, 25, 125, 625, and 3125 is

(a) 5

(b) 25

(c) 125

(d) 625

(e) 3125

**Q8.** The range of 3, 0, 1, -1, -3, 4, -5 is

(a) 0

(b) 3

(c) 9

(d) 10

(e) 12

**Q9.** The relation satisfied by the geometric mean 'G' of the numbers x, y, z (all positive) is

(a) G - xyz = 0

(b) G2 - xyz = 0

(c) G3 - xyz = 0

(d) G4 - xyz = 0

(e) None of these

**Q10.** The mode of the series 7, -7, -7, 0, -4, -7,2, 4, -7, -7, -4 is

(a) -11

(b) -7

(c) -4

(d) 0

(e) None of these

**Q11.** The median of first 15 composite numbers is

(a) 14

(b) 15

(c) 18.5

(d) 19

(e) 23

**Q12.** The mean deviation of the series 1, 3, 5, 7, 9, 11, 13 is

(a) 3.03

(b) 3.13

(c) 3.23

(d) 3.33

(e) 3.43

**Q13.** For a moderately skewed distribution, which of the following relationship holds?

(a) Mean-Mode = 3( Mean- Median)

(b) Median-Mode = 3(Mean-Median)

(c) Mean-Median =3( Mean-Mode)

(d) Mean-Median=3(Median-Mode)

**Q14.** Which of the following results holds for a set of distinct positive observations?

(a) AM≥GM HM

(b) HM≥GM AM

(c) AM>GM HM

(d) GM>AM>HM

**Q15.** If there are two groups containing 30 and 20 observations and having 50 and 60 as their arithmetic means, then the combined arithmetic mean is

(a) 55

(b) 56

(c) 54

(d) 52

**Q16.** If there are two groups with 75 and 65 as harmonic means and containing 15 and 13 observations, then the combined HM is given by

(a) 65

(b) 70.36

(c) 70

(d) 71

**Q17.** An aeroplane flies from A and B at the rate of 500km/hr and comes back from B to A at the rate of 700km/hr. The average speed of the aeroplane is

(a) 600 Km/hr

(b) 583.33 Km/hr

(c) 100 Km/hr

(d) 620Km/hr

**Q18.** If the AM and GM for 10 observations are both 15, then the value of HM is

(a) Less than 15

(b) More than 15

(c) 15

(d) Cannot be determined.

**Q19.** If the AM and GM for two numbers are 6.5 and 6 respectively then the two numbers are

(a)6 and 7

(b) 9 and 4

(c) 10 and 3

(d) 8 and 5

**Self Study**

**Chapter 8**

|  |
| --- |
| Trigonometry |

|  |
| --- |
| 8.1 Learning objective : |

To learn the basics of trigonometry.

|  |
| --- |
| 8.2 Basic formulae: |

**8.2.1 Basic t-ratios**

Hypotenuse1

Opposite

Adjacent

**8.2.2 Table of values**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **00** | **300** | **450** | **600** | **900** |
| **Sin** | 0 |  |  |  | 1 |
| **Cos** | 1 |  |  |  | 0 |
| **Tan** | 0 |  | 1 |  |  |

**8.2.3 Trigonometry Identities**

1. Sin2 θ + Cos2 θ = 1

2. Sec2 θ - Tan2 θ = 1

3. Cosec2 θ - Cot2 θ = 1

**8.2.4 Multiple Angle Identities**

1. Sin 2A = 2 Sin A. Cos A

=

2. Cos 2A = 1 - 2Sin2 A

= 2Cos2 A - 1

=

3. Tan 2A =

4. Sin2 A =

5. Cos2 A =

6. Sin 3A = 3 Sin A - 4 Sin3 A

7. Cos 3A = 4 Cos3 A - 3 Cos A

8. Tan3A=

**8.2.5 Sub-multiple angle formula**

1. Sin A = 2 Sin (A/2) Cos (A/2)

2. Cos A = 1 - 2Sin2

= 2Cos2 - 1

3.

**8.2.6 Half-angle Formula**

1. Sin =

2. Cos =

3. Tan =

**8.2.7 Product formula**

1. Sin C + Sin D = 2 Sin

2. Sin C - Sin D = 2 Cos

3. Cos C + Cos D = 2 Cos 2]Cos

4. Cos C - Cos D = - 2 Sin

Let

A

B

C

B

A

C

c

b

a

**8.2.8 Heron’s formula / Lami’s theorem / Sine formula**

Where R is the circum radius of the

triangle.

**8.2.9 Cosine formula**

a2 = b2 + c2 – 2bcCos(A)

|  |
| --- |
| * 1. Solved Examples |

**Q1.** If sin x + sin2 x = 1, then cos2 x + cos4 x =

(a) 0

(b) 1

(c) 1.5

(d) 2

Correct option:(b)

**Explanatory answer:**

sin x + sin2 x = 1 [Given]

⇒ sin x = 1 - sin2 x = cos2 x

∴ cos2 x + cos4 x = sin x + sin2 x = 1

**Q2.** Tan (cos-1 x) =

Correct answer option is (c)

**Explanatory answer:**

Let cos-1 x = y

Cos y = x

Cos y =

y

1

x

Tan y =

Tan y =

Tan (cos-1 x) = Tan (y)

=

**Q3**.

(a) 0

(b) 1

(c)

(d)

(e)

Correct option: (d)

**Explanatory answer:**

Using sin(π - θ) = sin θ,

sin(9π ⁄ 14) = sin(5π ⁄ 14)

sin(11π ⁄ 14) = sin(3π ⁄ 14)

sin(13π ⁄ 14) = sin(π ⁄ 14)

Also, sin(7π ⁄ 14) = sin(π ⁄ 2) = 1

∴ =

       =

       [Using sin θ = cos(π ⁄ 2 - θ)

=

**Q4.** Number of solutions of the equation tan x + sec x = 2 cos x, lying in the interval [0, 2π] is

(a) 0

(b) 1

(c) 2

(d) 3

Correct option: (c)

**Explanatory answer:**

tan x + sec x = 2 cos x

sin x + 1 = 2 cos2 x, cos x ≠ 0

sin x + 1 = 2 (1 - sin2 x)

2 sin2 x + sin x - 1 = 0

This is quadratic equation in sin x.

Solve for sin x

sin x = - 1, or sin x = ½

sin x = -1 ⇒ cos x = 0,

which is not possible

[cos x ≠ 0 from above]

∴ sin x = ½

x = π or x = π - (π ⁄ 6), in the interval [0, 2π]

x = π or x = 5π ⁄ 6

|  |
| --- |
| 8.4 Problems for Practice |

**Q1.** For ∆ABC, right angled at C, where AB=29, BC=21 and ∟B = 𝝓, what is the value for Cos2 𝝓 - Sin2 𝝓

**Q2.** In a right triangle ABC, right-angled at

B, if Tan A = 1, then evaluate

2 Sin A Cos A = ?

**Q3.** In ∆OPQ, OPQ, right-angled at P,

OP = 7 cm and OQ – PQ = 1 cm. Determine the value of Sin Q.

**Q4.** In ∆ABC, right-angled at B, AB = 5 cm and ∟ACB = 30°. Determine the length of the sides BC.

**Q5.** If Sin (A – B) = ½, Cos (A + B) = ½ where 0° < A + B 90°, A > B, find ∟A ?

**Q6**. What is the value of

Tan 65°/ Cot 25° ?

**Q7**. If Sin 3A = Cos (A – 26°), where 3A is an acute angle, find the value of ∟A ?

**Q8**. If sec 4A = Cosec (A – 20°), where 4A is an acute angle, find the value of ∟A ?

**Q9**. What is the value of

Sec A (1-Sin A) (Sec A + Tan A) ?

**Q10**. What is the value of

9 Sec2A – 9 Tan2A ?

**Q11**. What is the value of

(sec A + Tan A) (1 – Sin A) =?

(a) sec A

(b) Sin A

(c) Cosec A

(d) Cos A

**Q12.** What is the value of